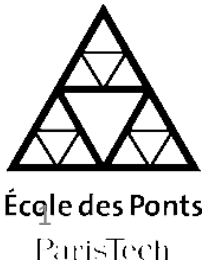
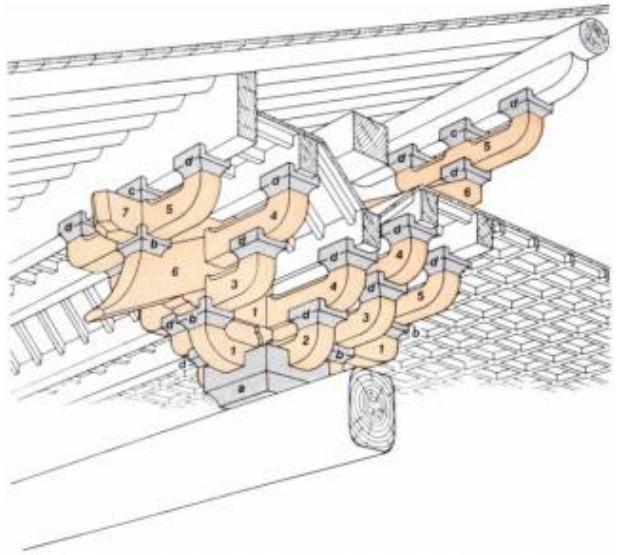
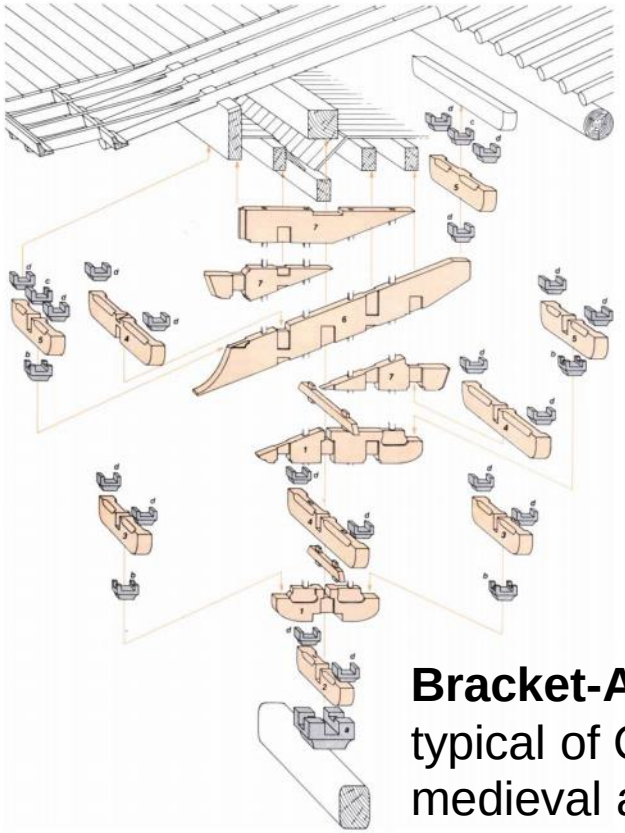
Generative design of interlocking sequential assemblies

Pierre Gilibert

Supervisors: O. Baverel and R. Mesnil



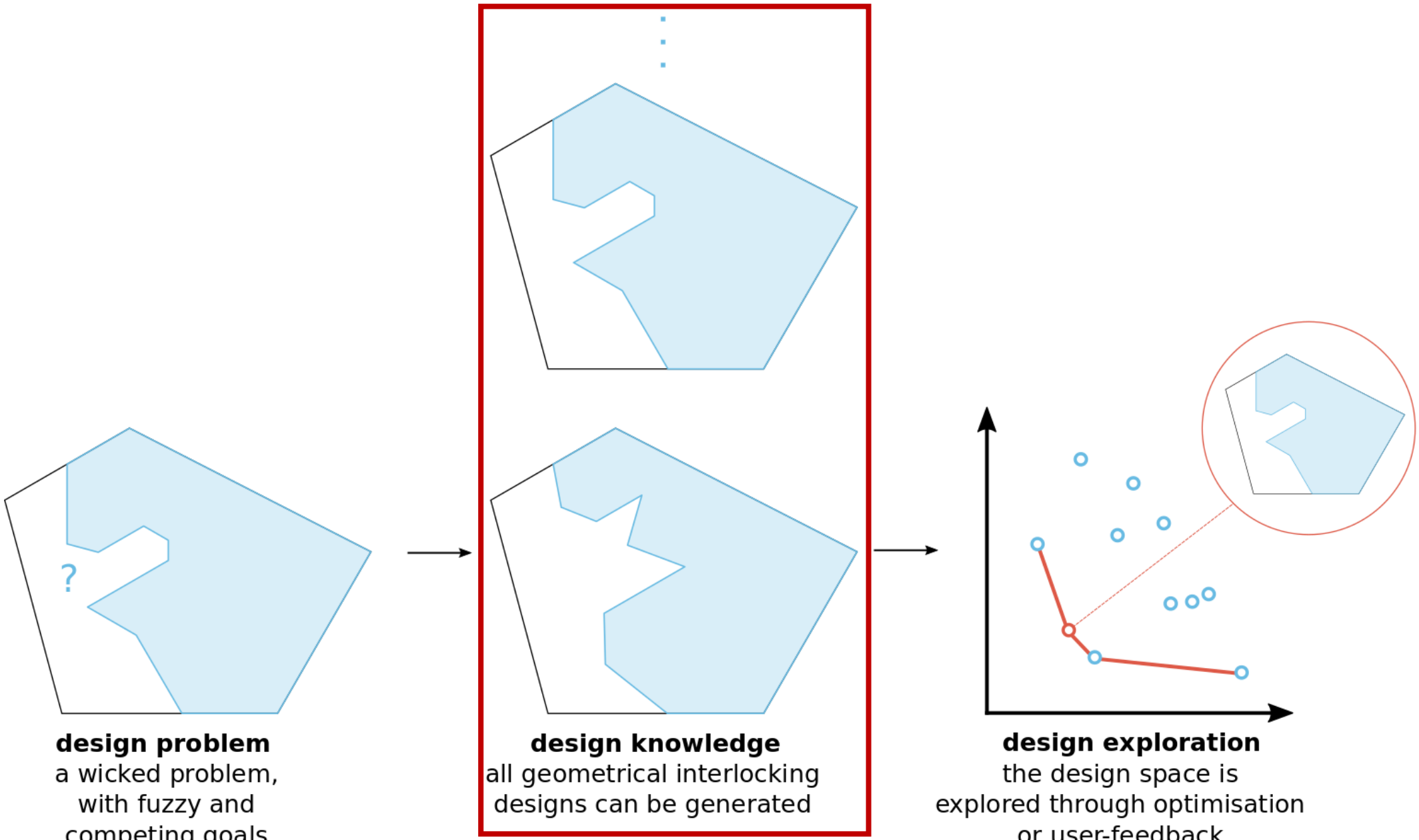




Bracket-Arm Set
typical of Chinese
medieval architecture

Hoshi Ryokan inn inaugurated in 719, Japan

Focus on the assembly node



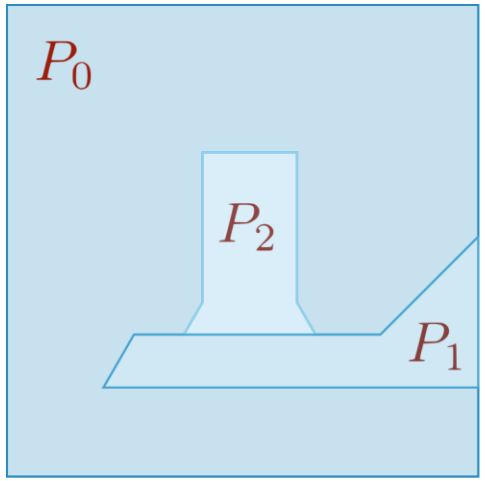
design problem
a wicked problem,
with fuzzy and
competing goals

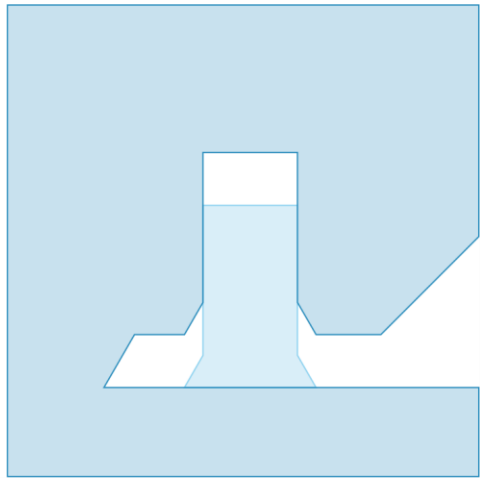
design knowledge
all geometrical interlocking
designs can be generated

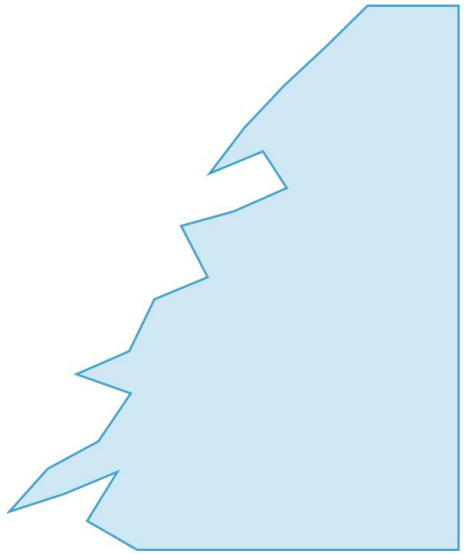
design exploration
the design space is
explored through optimisation
or user-feedback

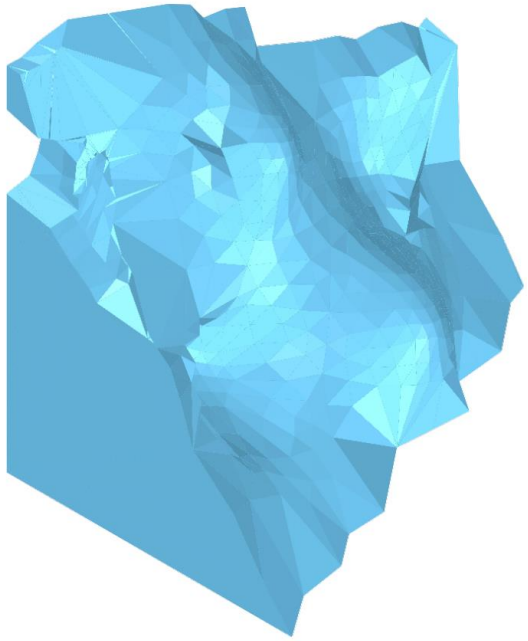
Simplifying assumptions:

- Infinitesimal motions
- Polygonal/Polyhedral parts

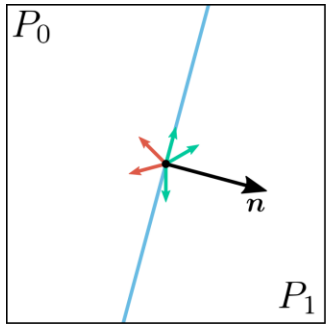


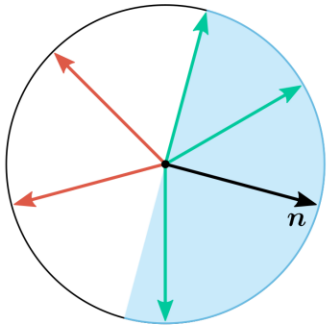


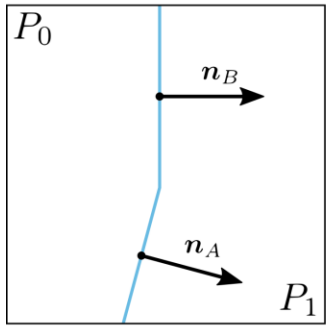


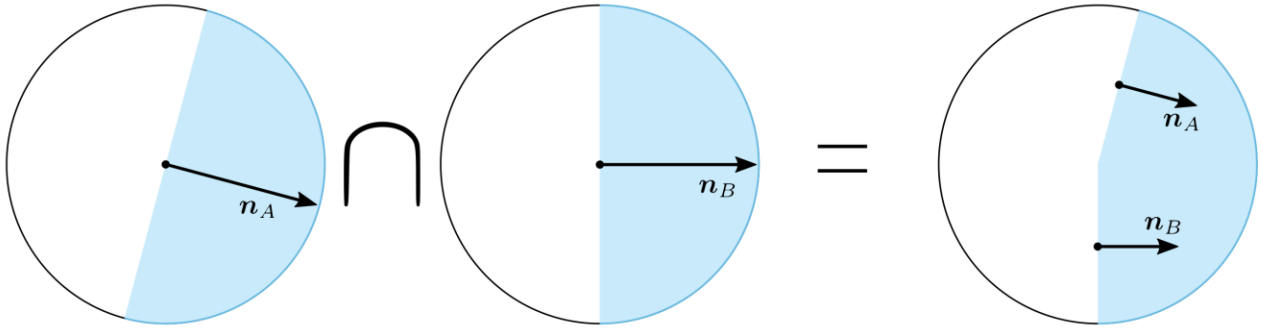


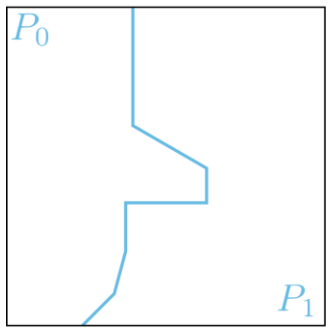
2D - Theory

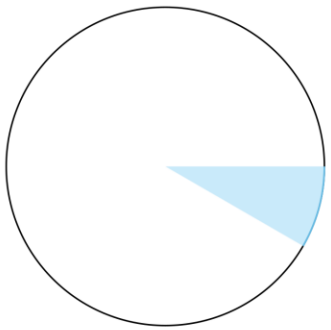




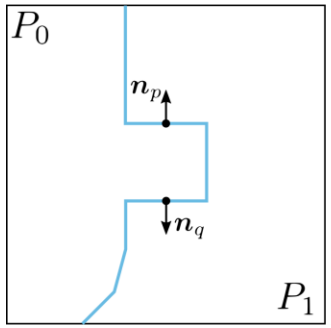


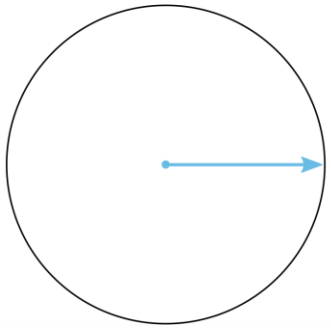




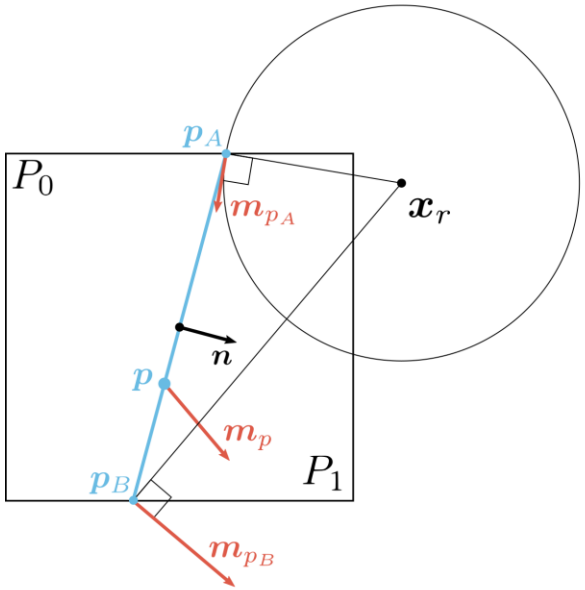


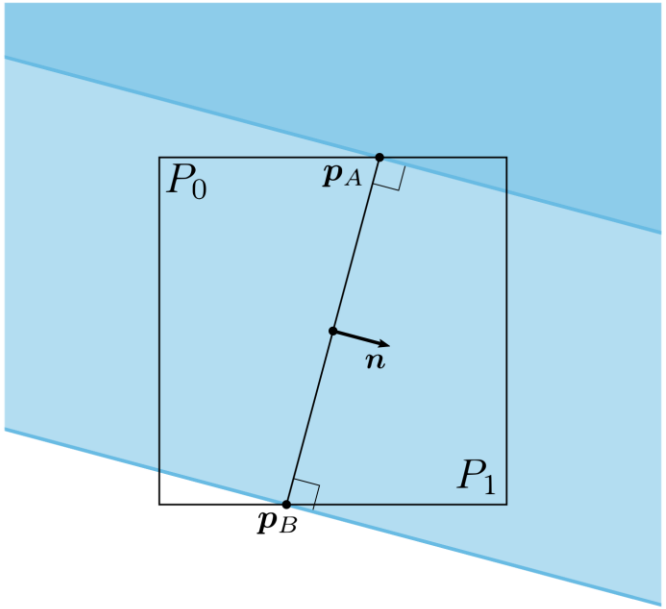
Cone of translational freedom

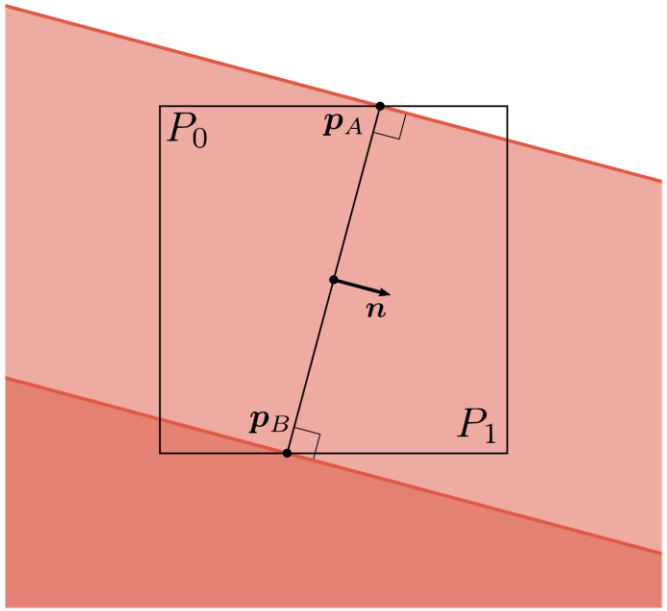


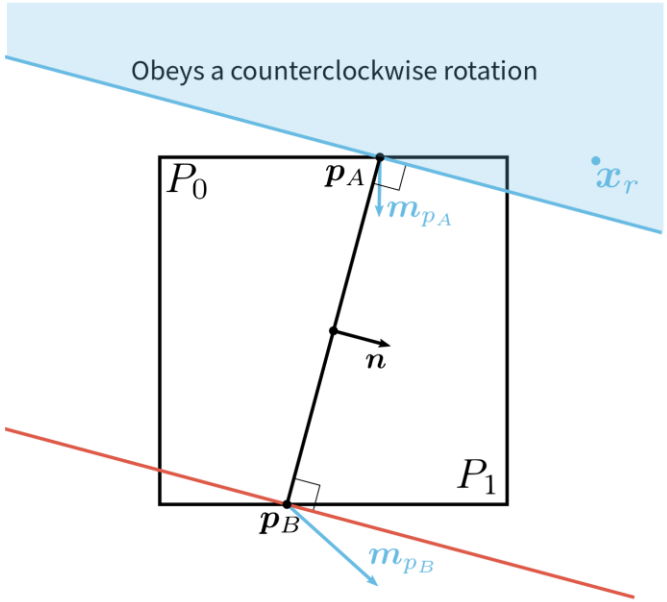


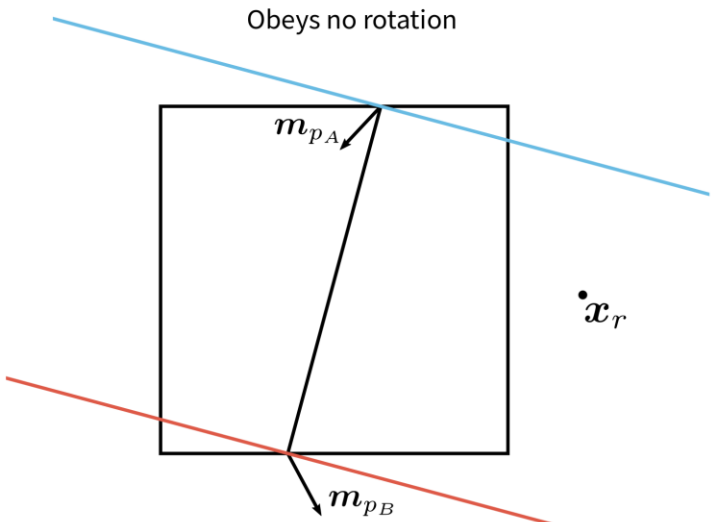
Snap

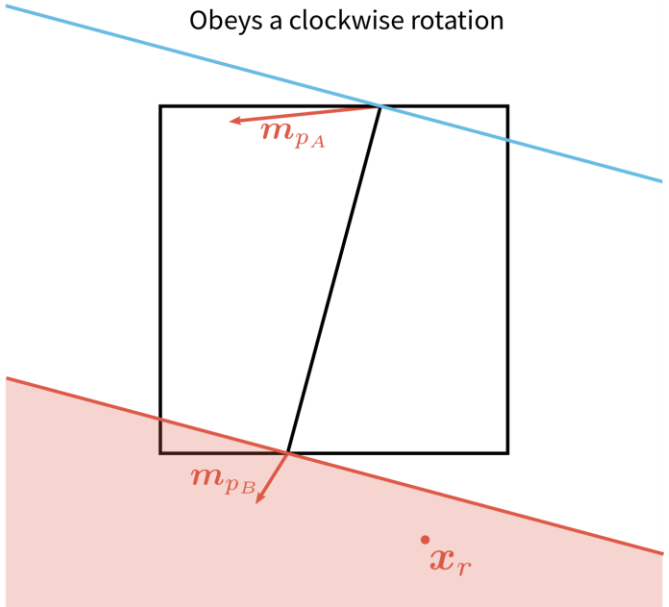




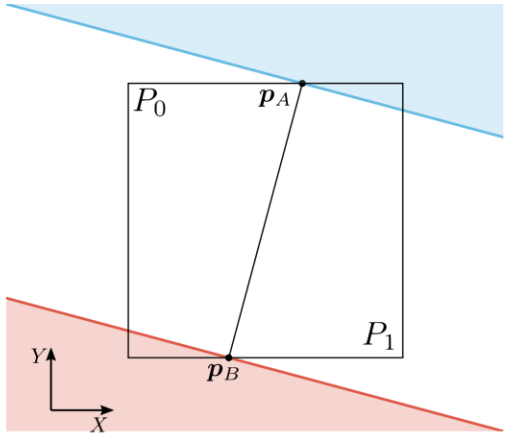


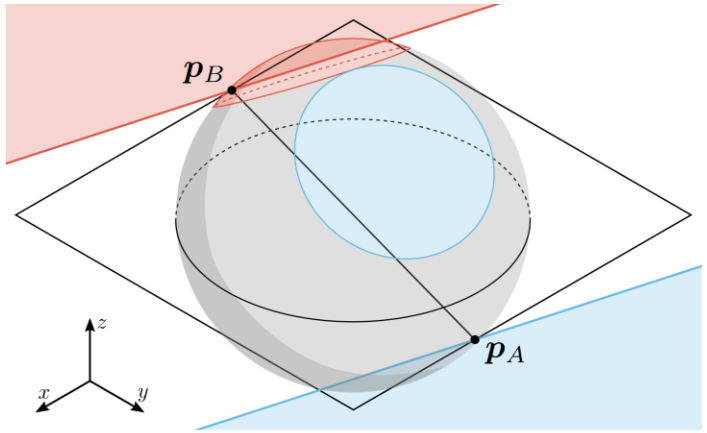


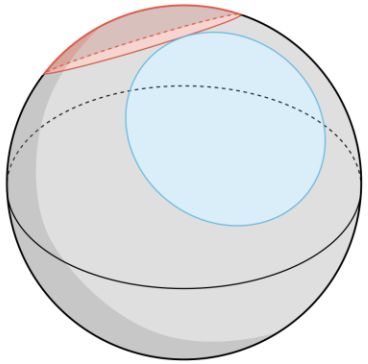


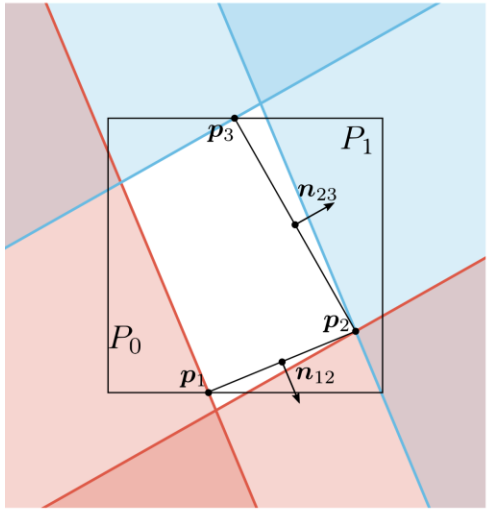


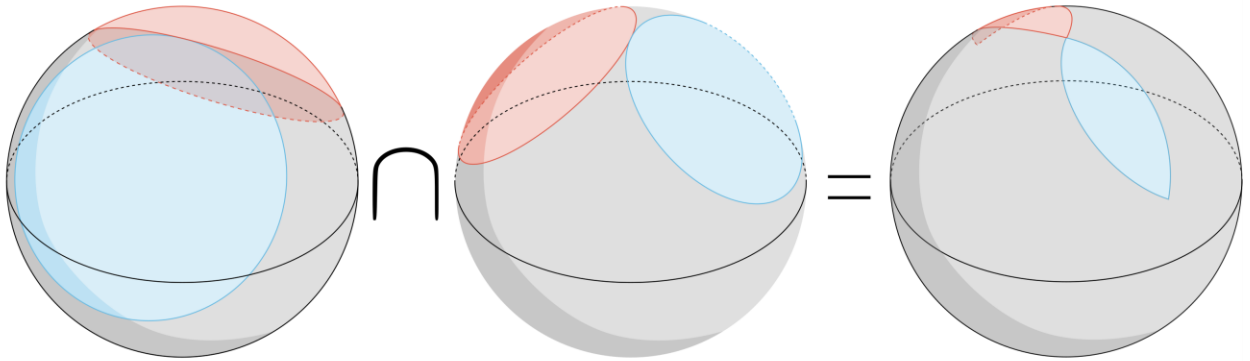
Half-planes of rotational freedom



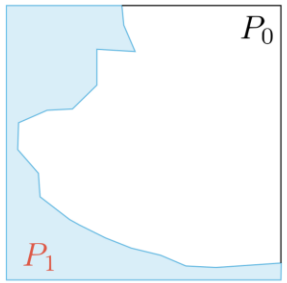


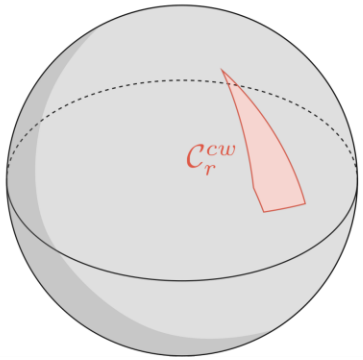


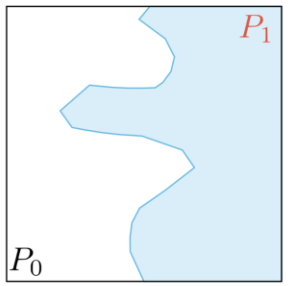


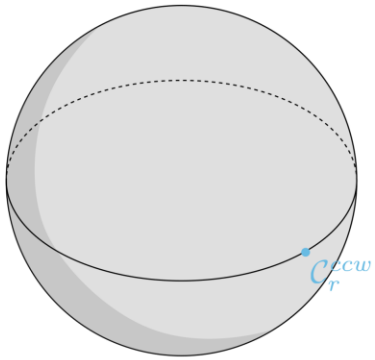


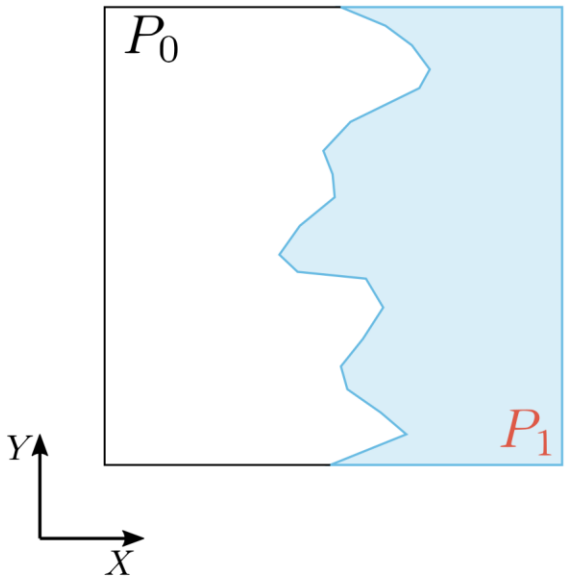
Cones of rotational freedom

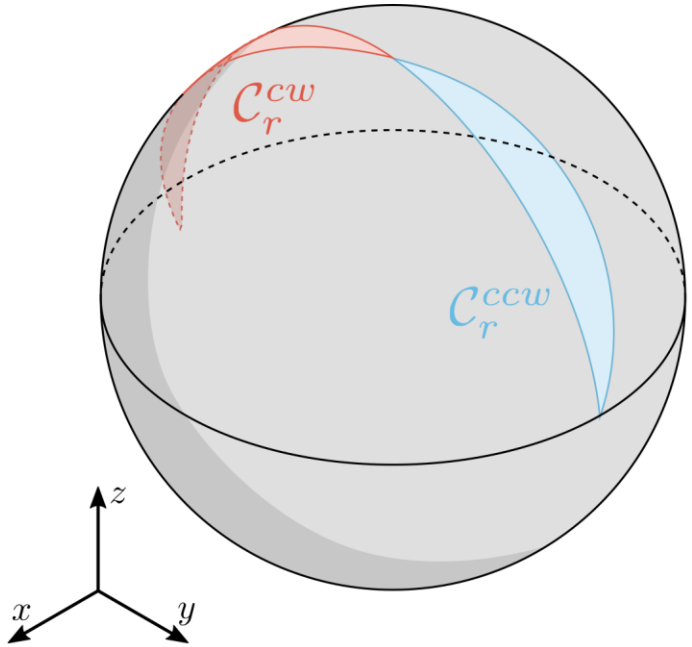


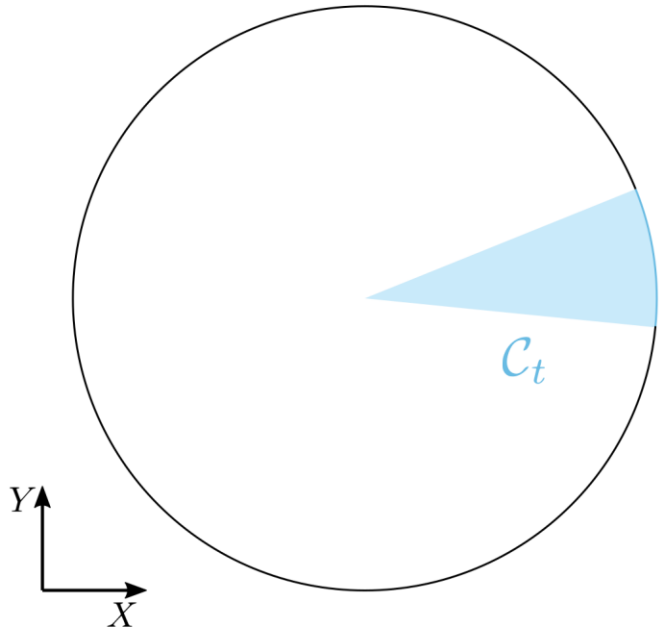




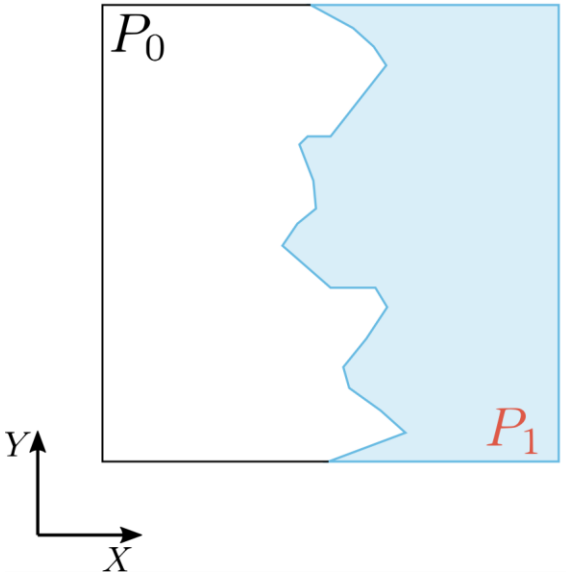


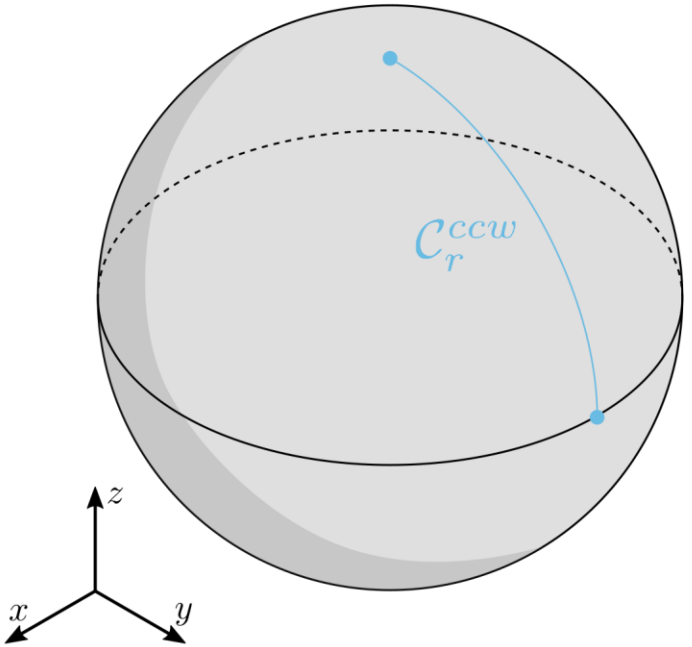


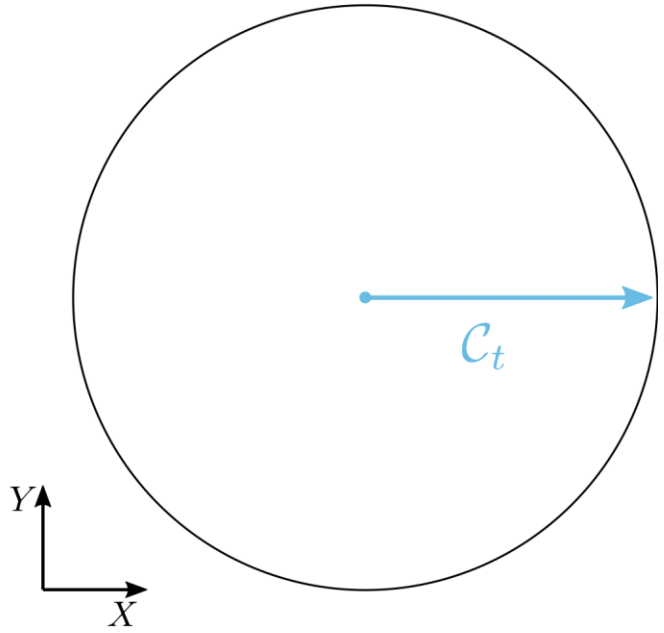


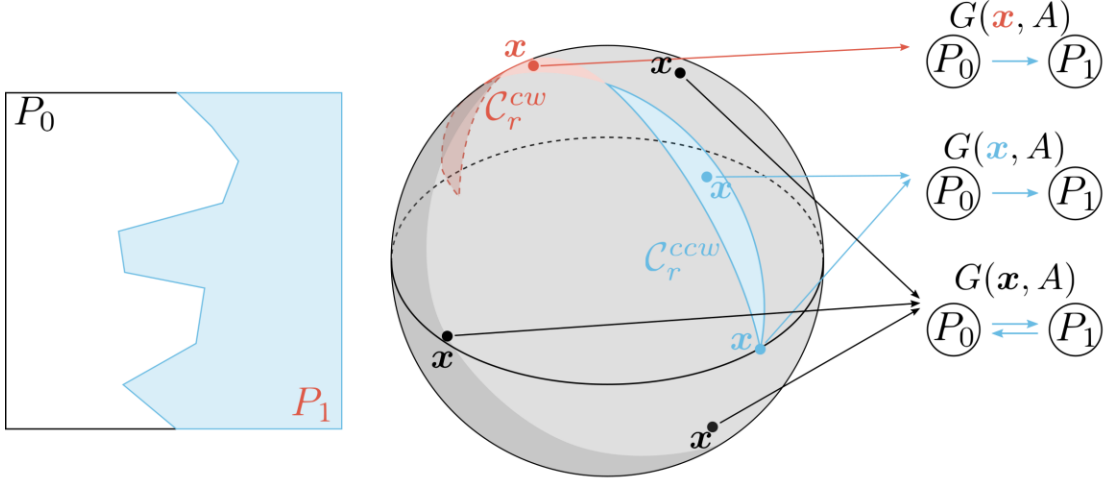


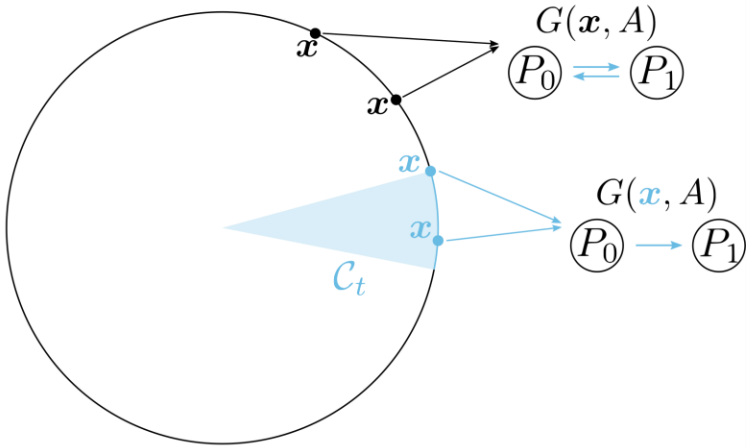
Cones of freedom

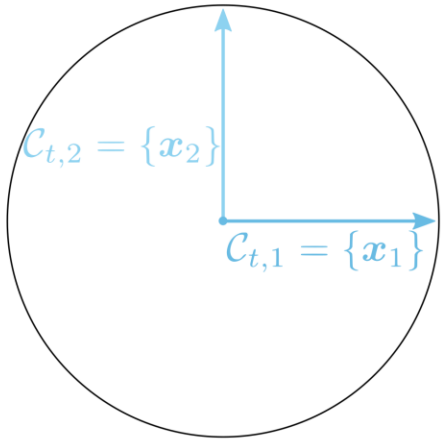


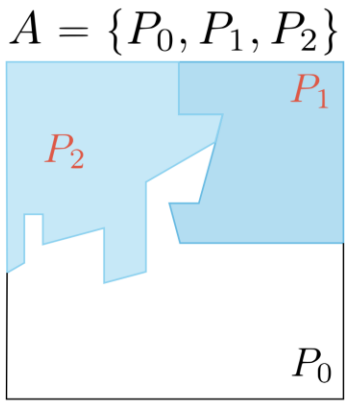


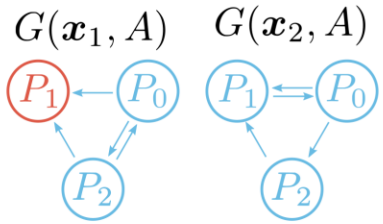


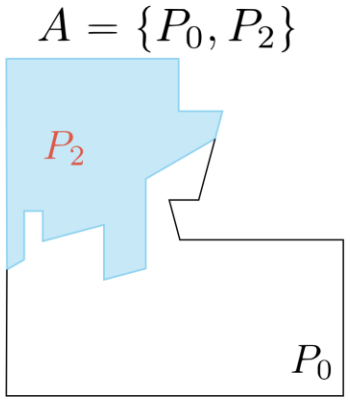


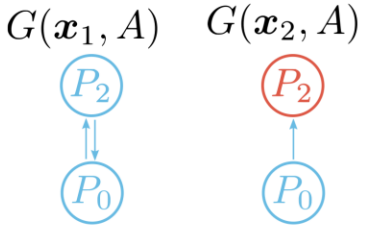




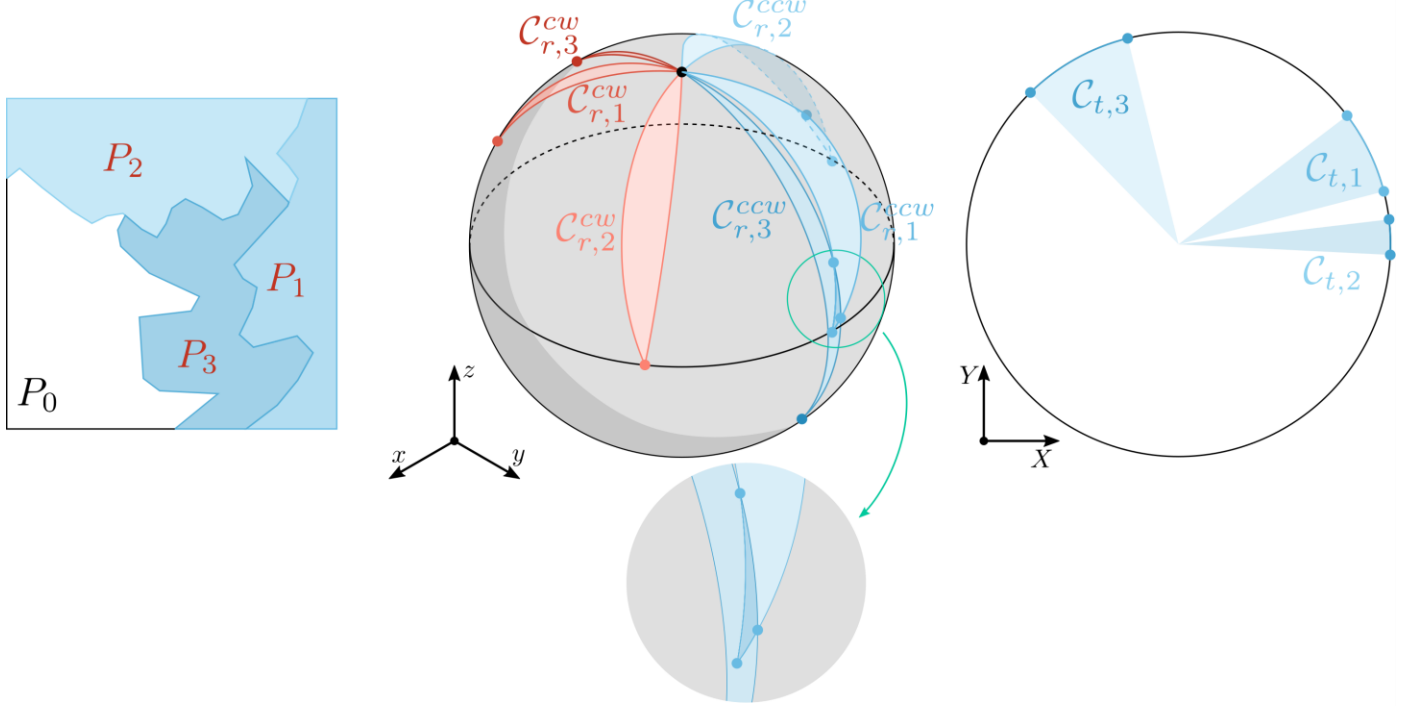


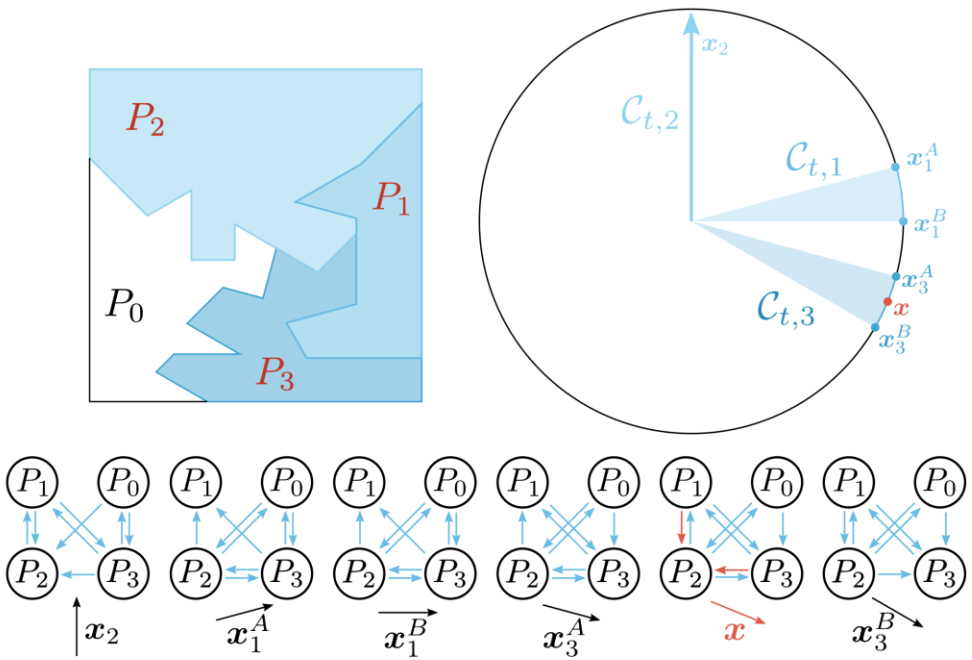






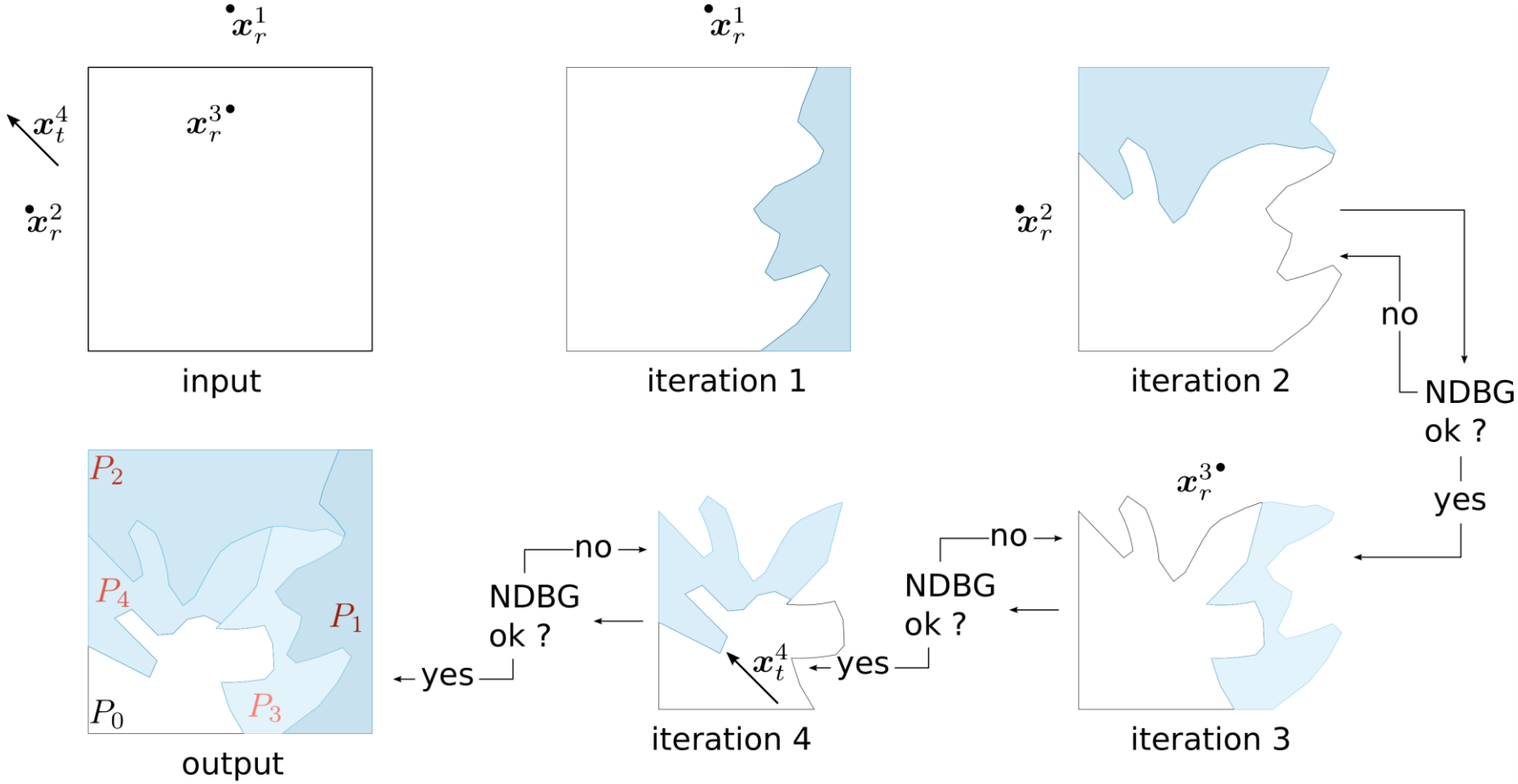
Directional Blocking Graph (DBG)

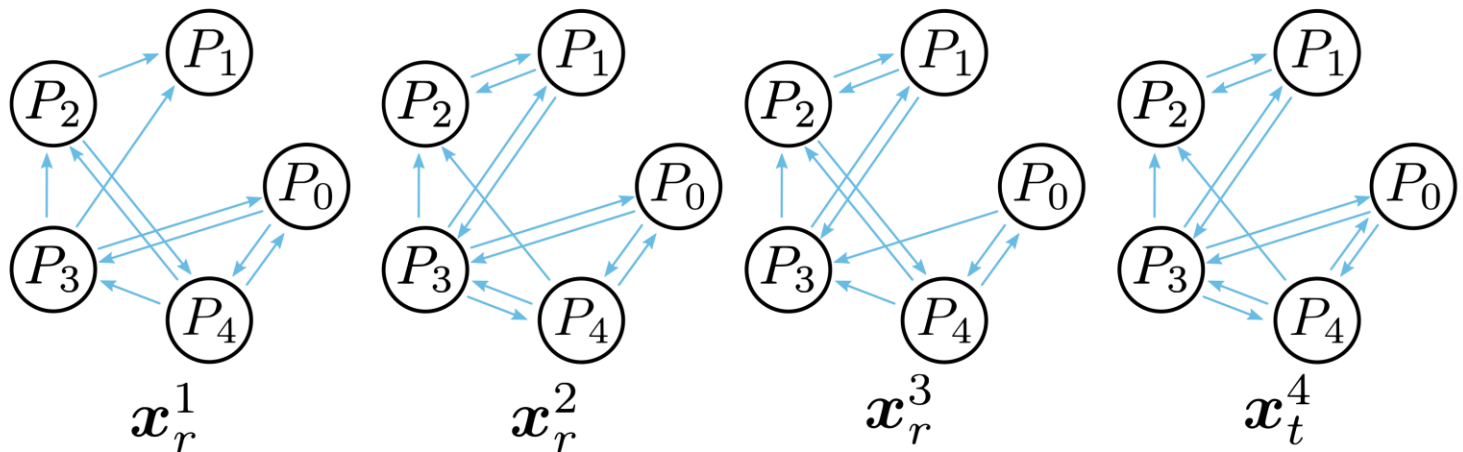




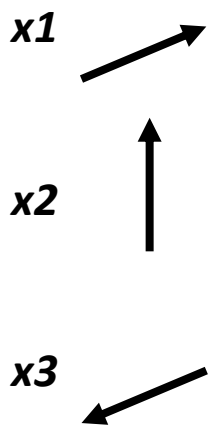
Non Directional Blocking Graph (NDBG)

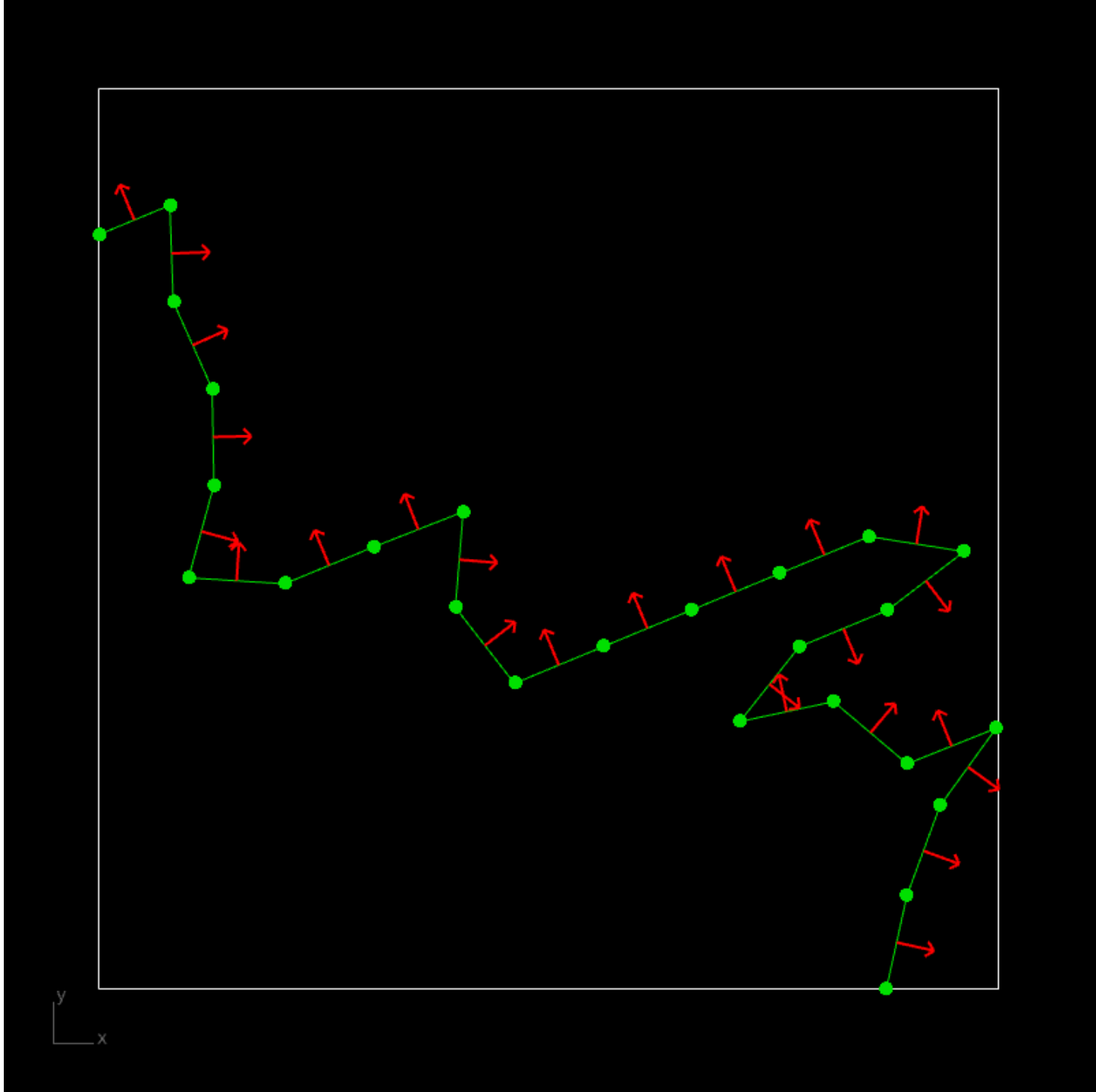
2D - Implementation


x_r^1
x_t^4
x_r^3
x_r^2
input
iteration 1
iteration 2
no
NDBG
ok ?
yes
P_2
P_4
P_1
P_0
P_3
output
no
NDBG
ok ?
yes
iteration 4
iteration 3

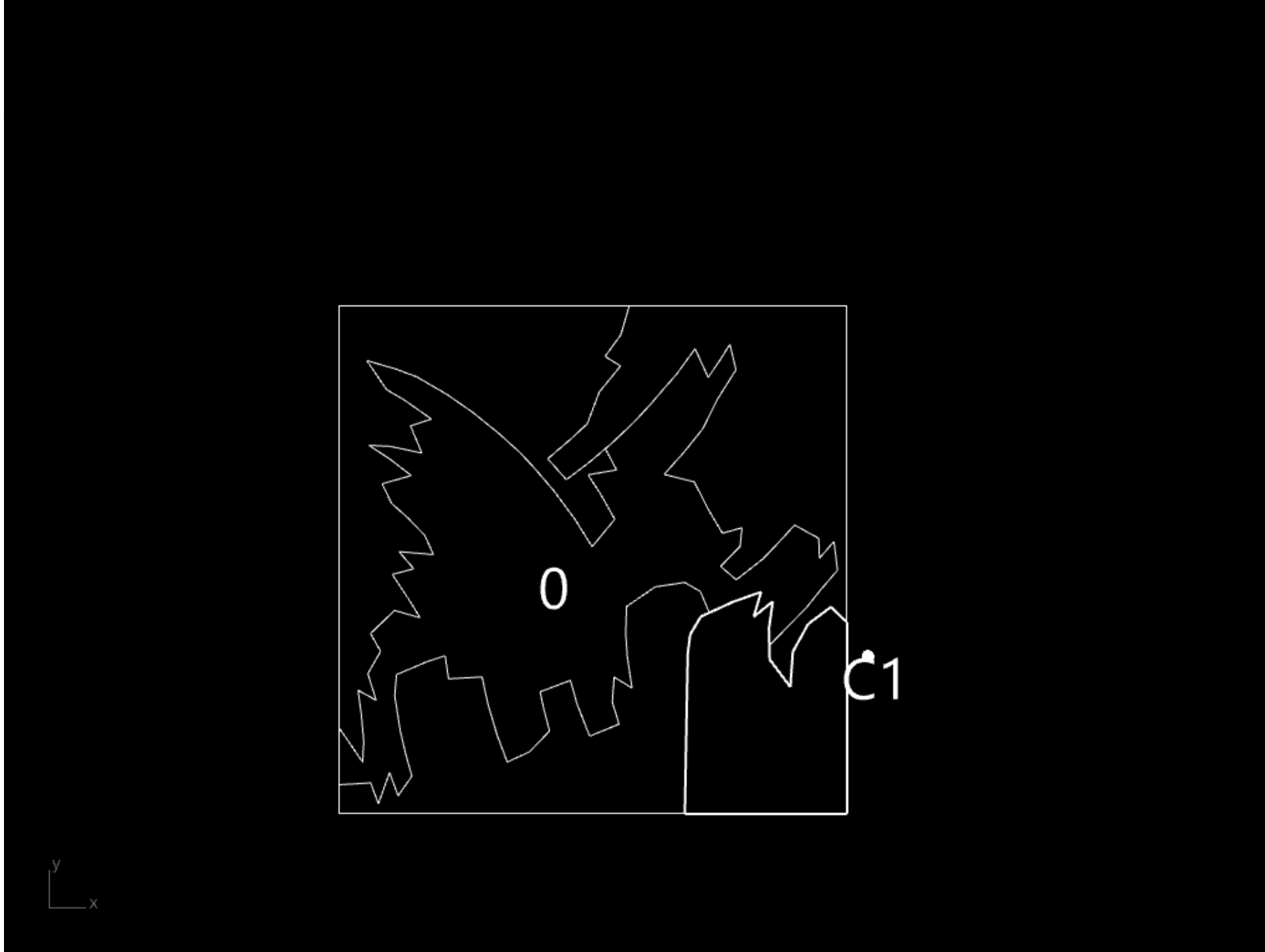

P_1
P_2
P_0
P_3
P_4
x_r^1
x_r^2
x_r^3
x_t^4

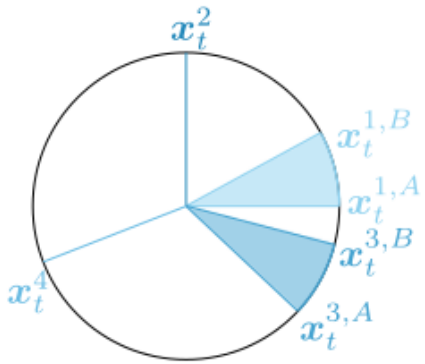
2D

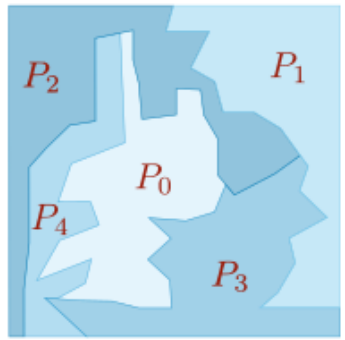


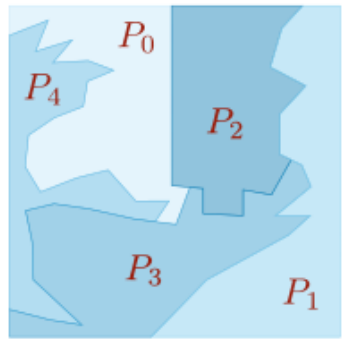


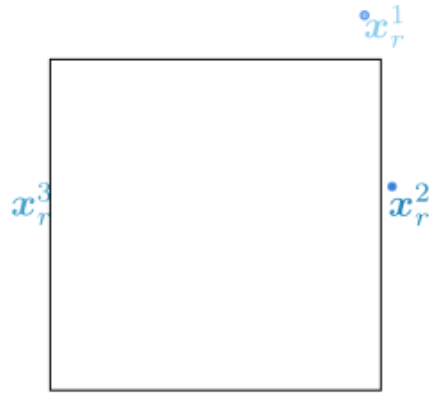
2D - rotat

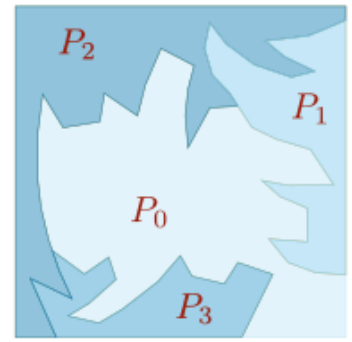


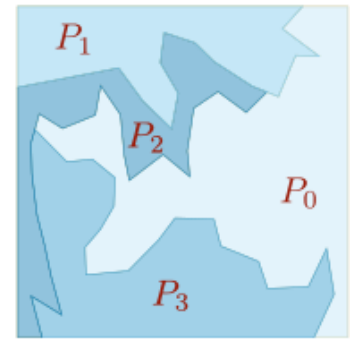

x_t^2
$x_t^{1,B}$
$x_t^{1,A}$
$x_t^{3,B}$
x_t^4
$x_t^{3,A}$

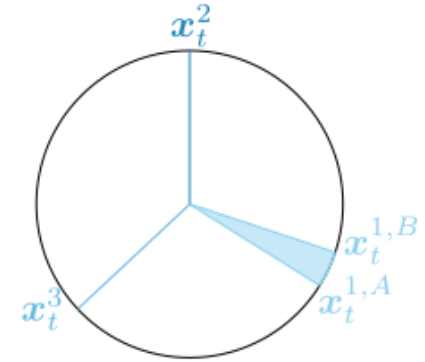

P_2
P_1
P_0
P_4
P_3

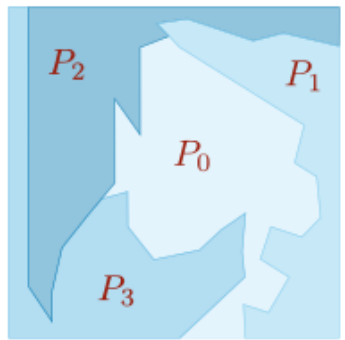

P_0
P_4
P_2
P_3
P_1

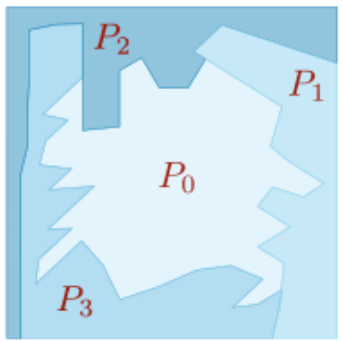

x_r^1
x_r^3
x_r^2

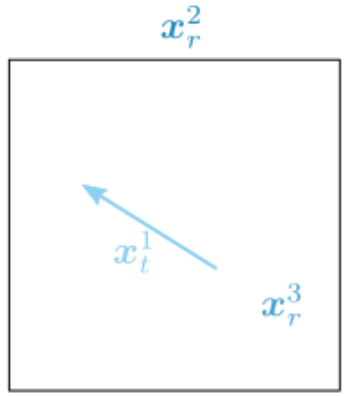

P_2
P_1
P_0
P_3

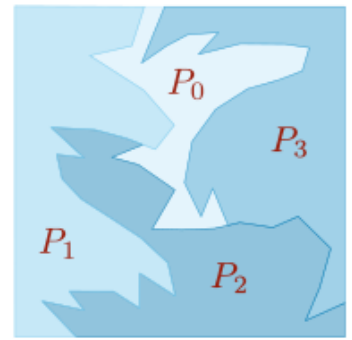

P_1
P_2
P_0
P_3

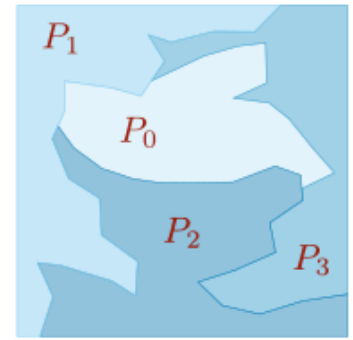

x_t^2
$x_t^{1,B}$
x_t^3
$x_t^{1,A}$

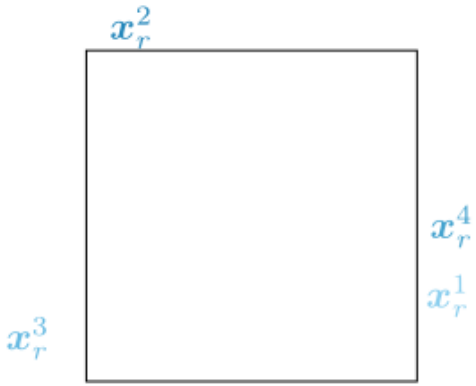

P_2
P_1
P_0
P_3

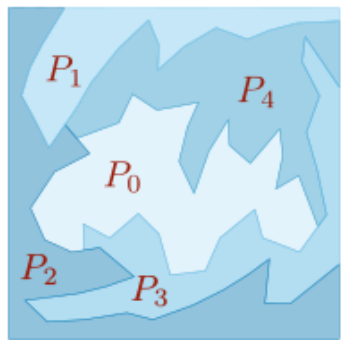

P_2
P_1
P_0
P_3

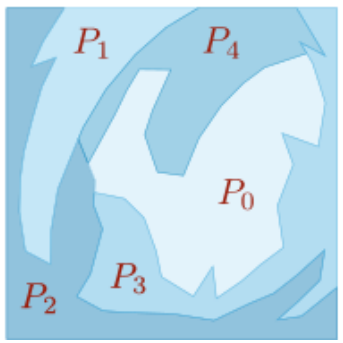

x_r^2
x_t^1
x_r^3
x_r^1


P_0
P_3
P_1
P_2


P_1
P_0
P_2
P_3

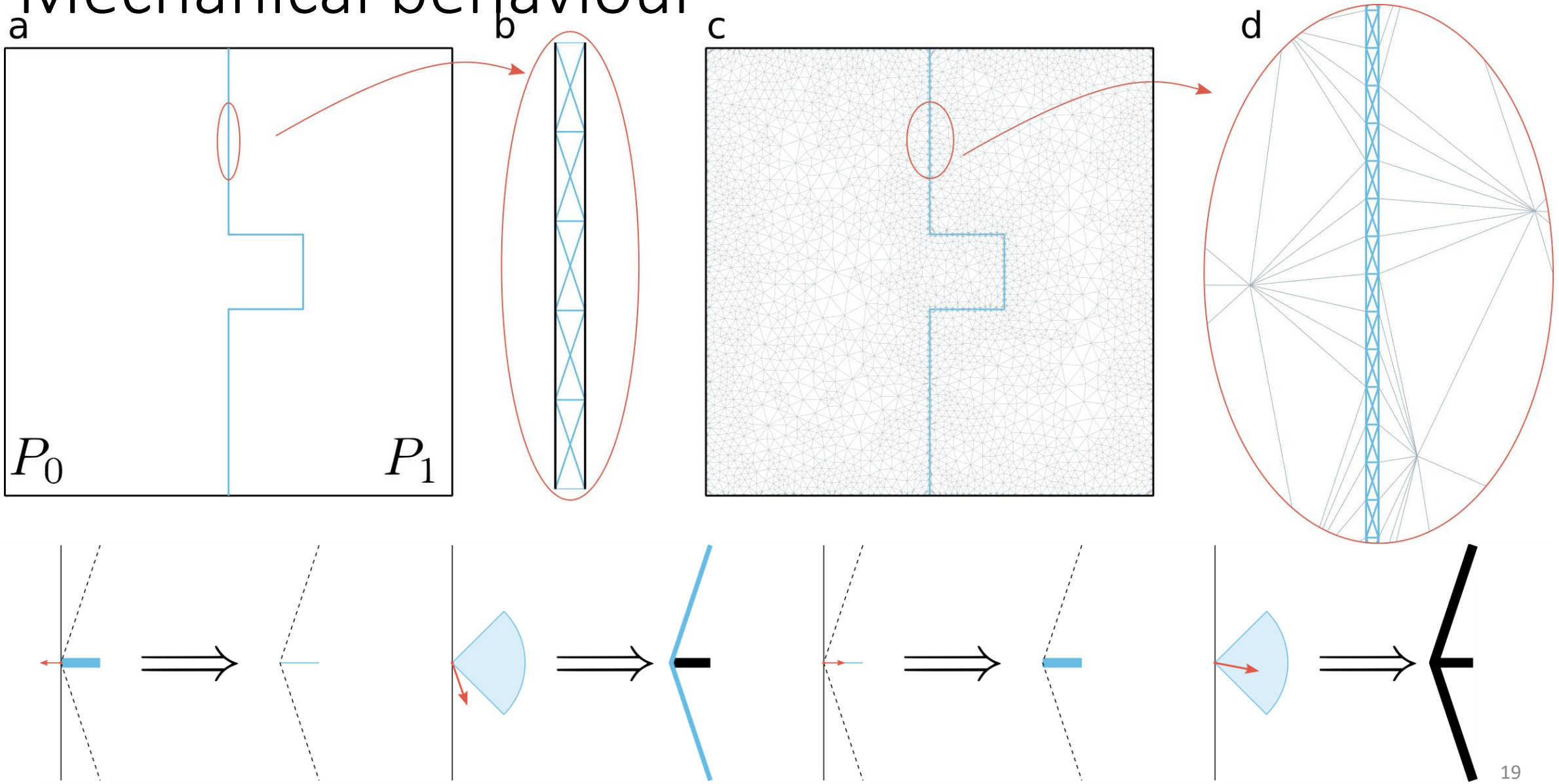

x_r^2
x_r^4
x_r^1
x_r^3

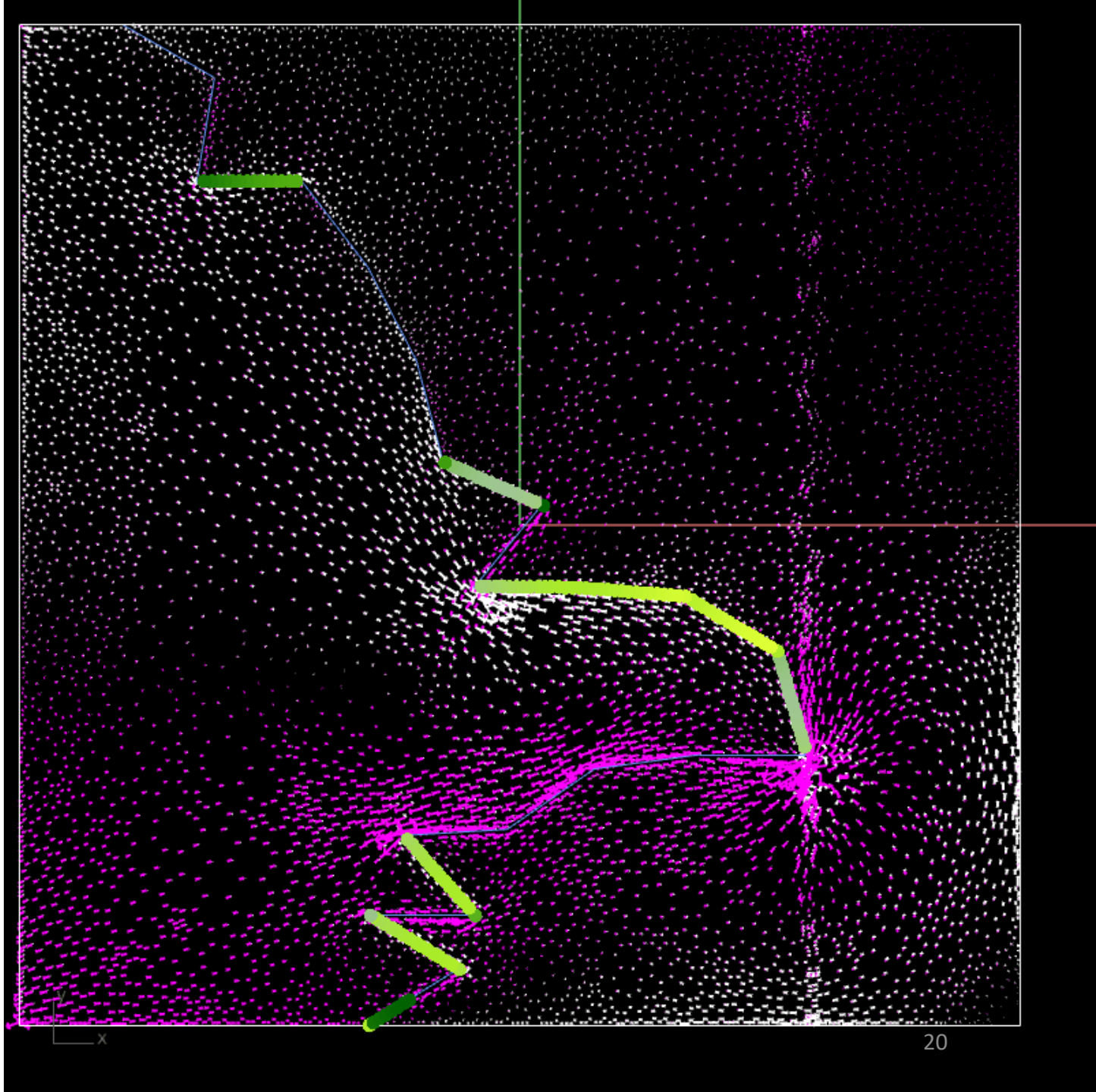

P_1
P_4
P_0
P_2
P_3

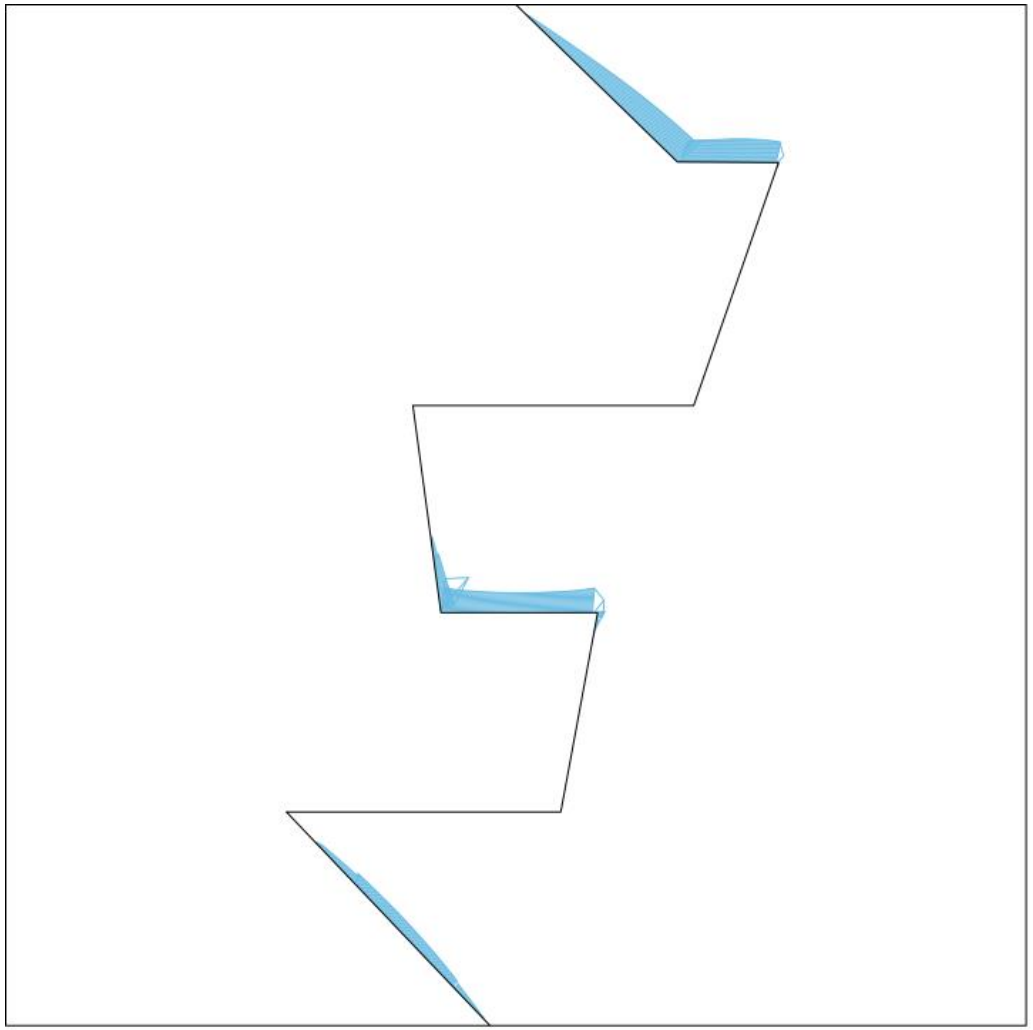

P_1
P_4
P_0
P_2
P_3

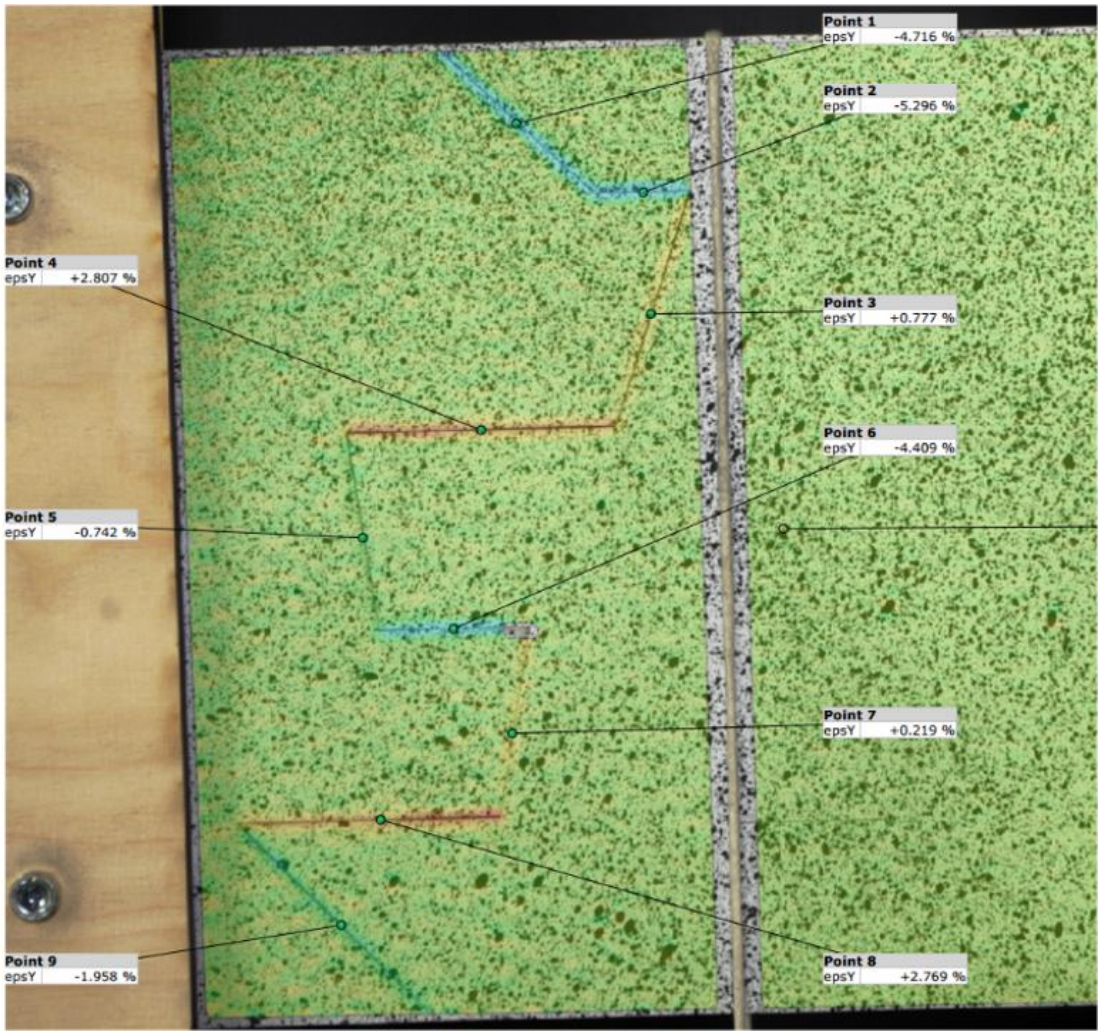
2D - Results

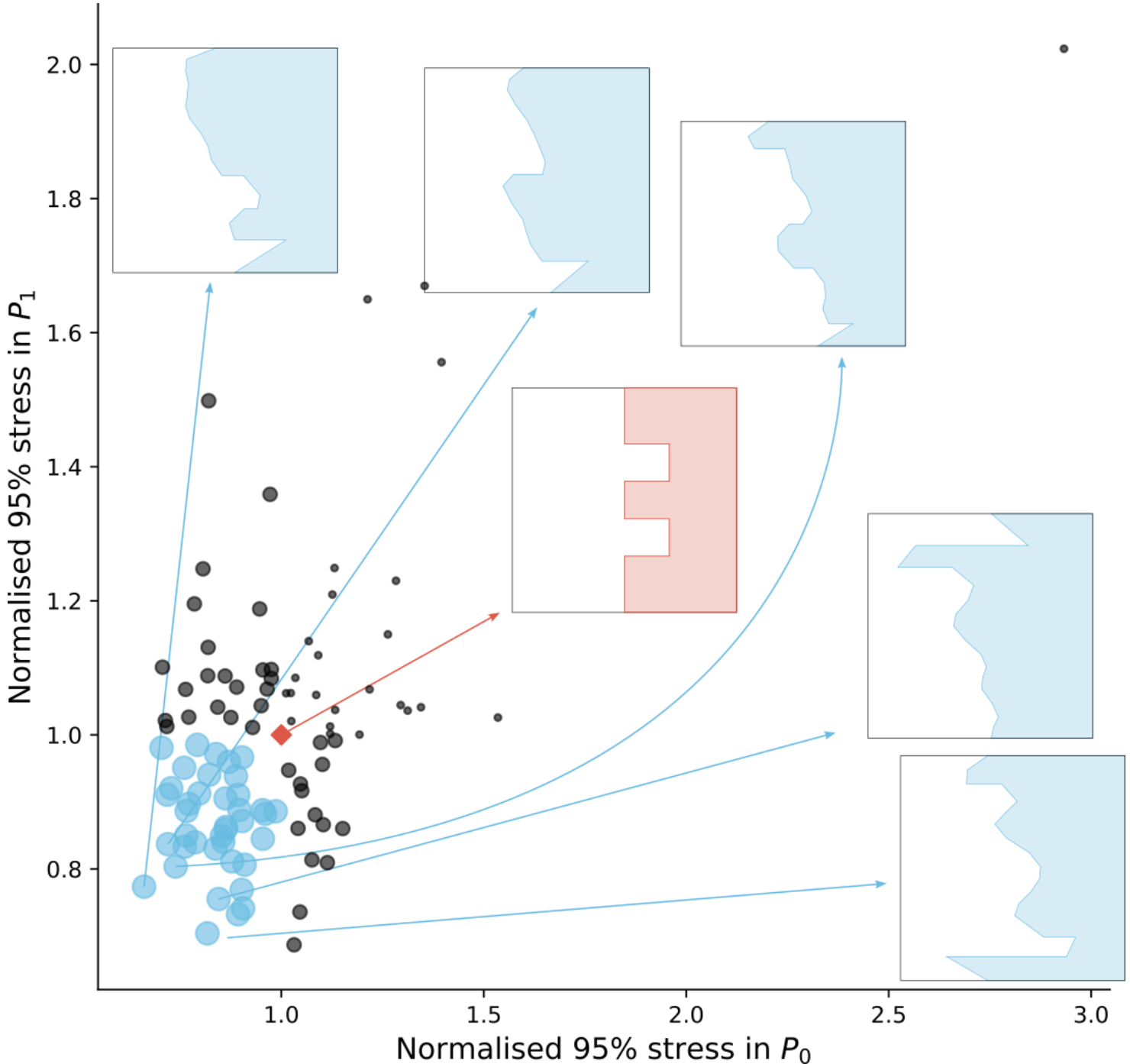
Mechanical behaviour




y
x

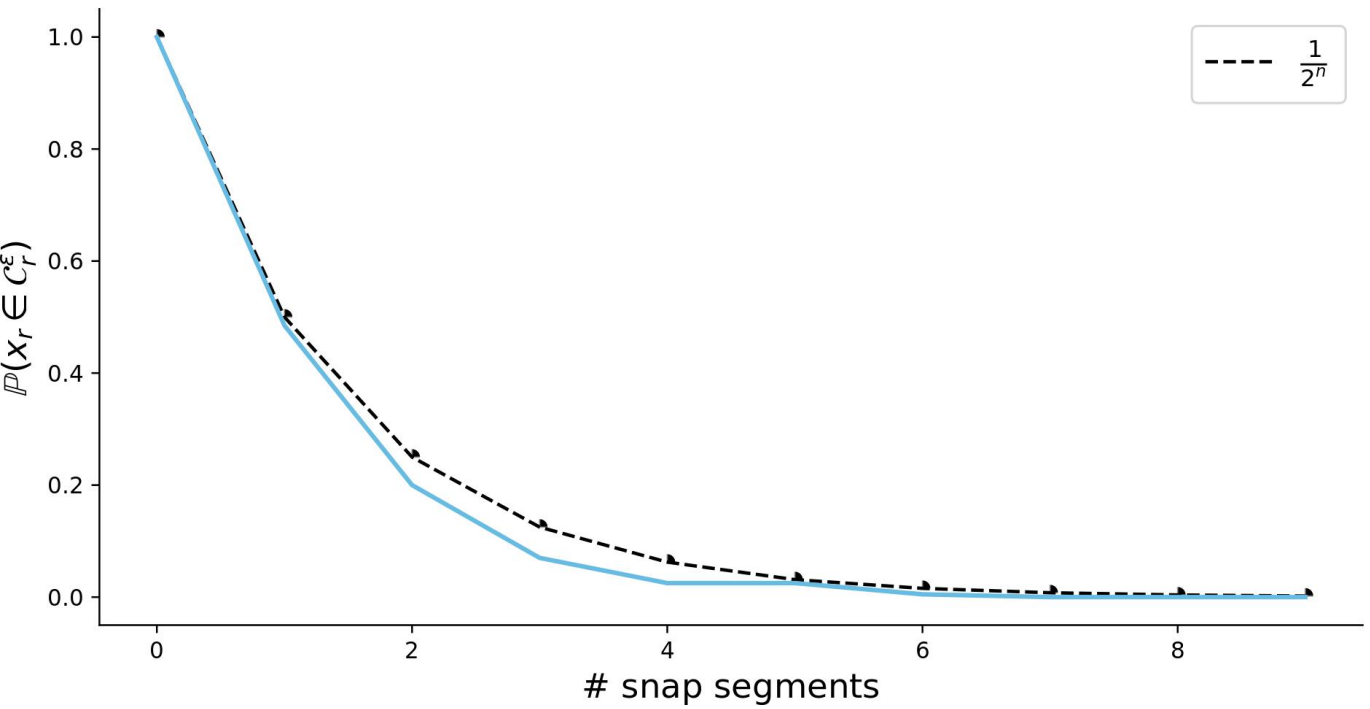


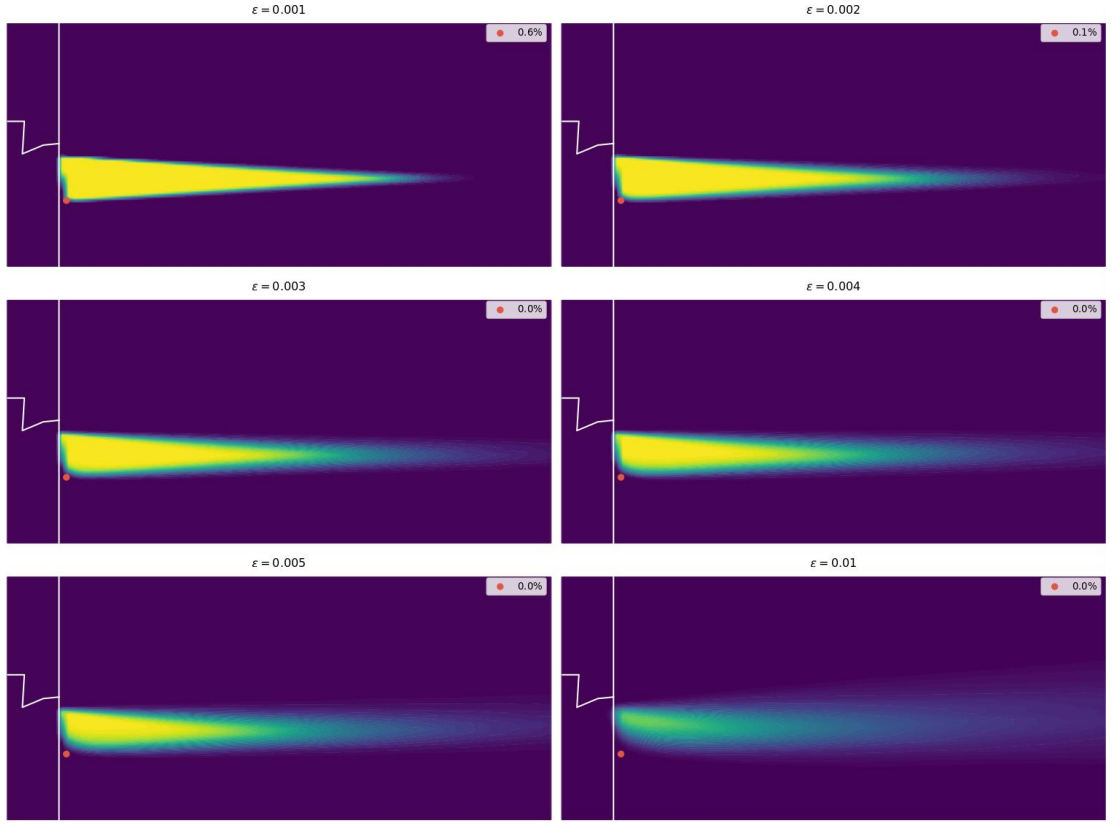

Point 1
epsY -4.716 %
Point 2
epsY -5.296 %
Point 4
epsY +2.807 %
Point 3
epsY +0.777 %
Point 6
epsY -4.409 %
Point 5
epsY -0.742 %
Point 7
epsY +0.219 %
Point 9
epsY -1.958 %
Point 8
epsY +2.769 %

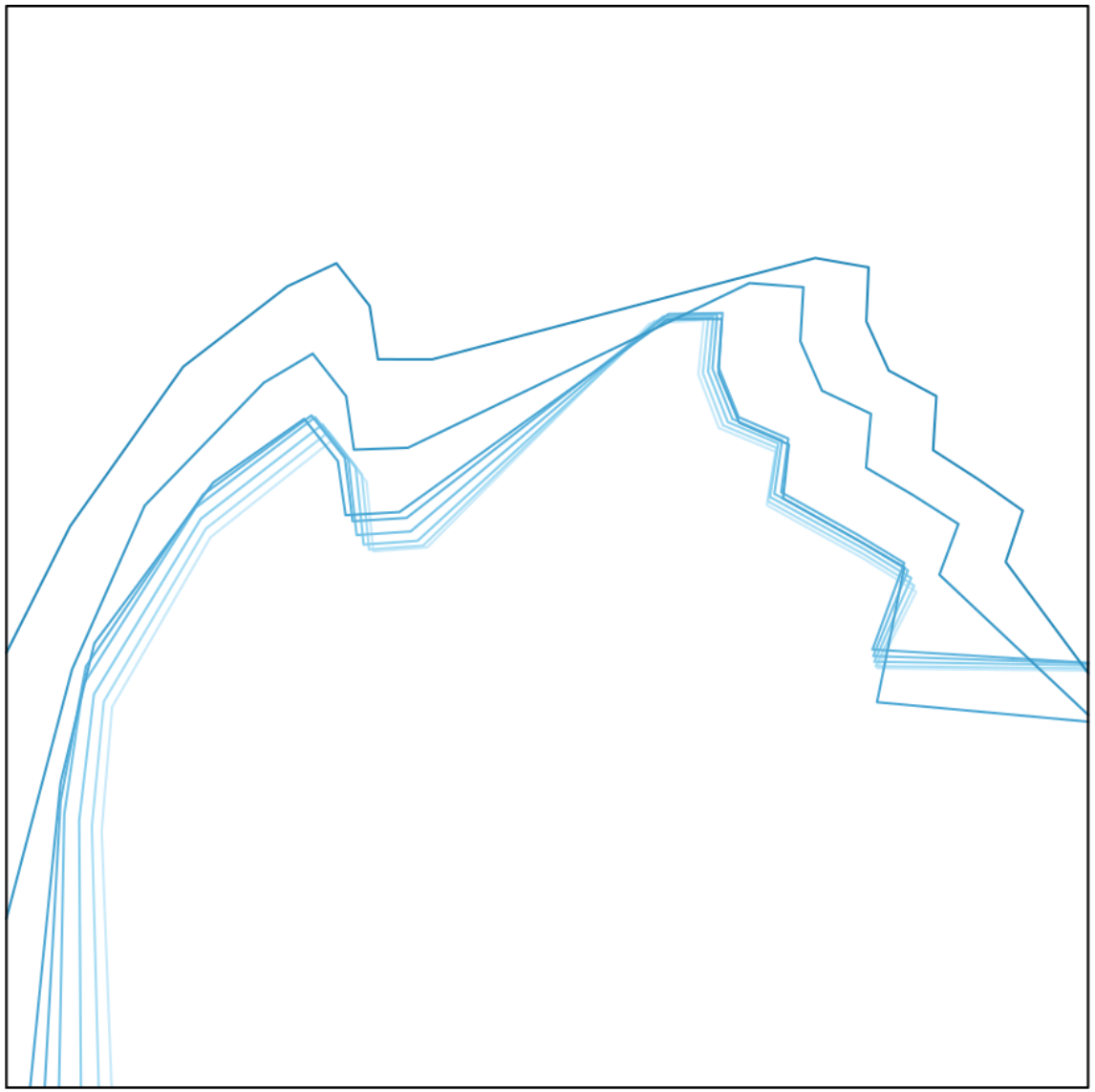

2.0
1.8
1.6
1.4
1.2
1.0
0.8
Normalised 95% stress in P_1
1.0
1.5
2.0
2.5
3.0
Normalised 95% stress in P_0

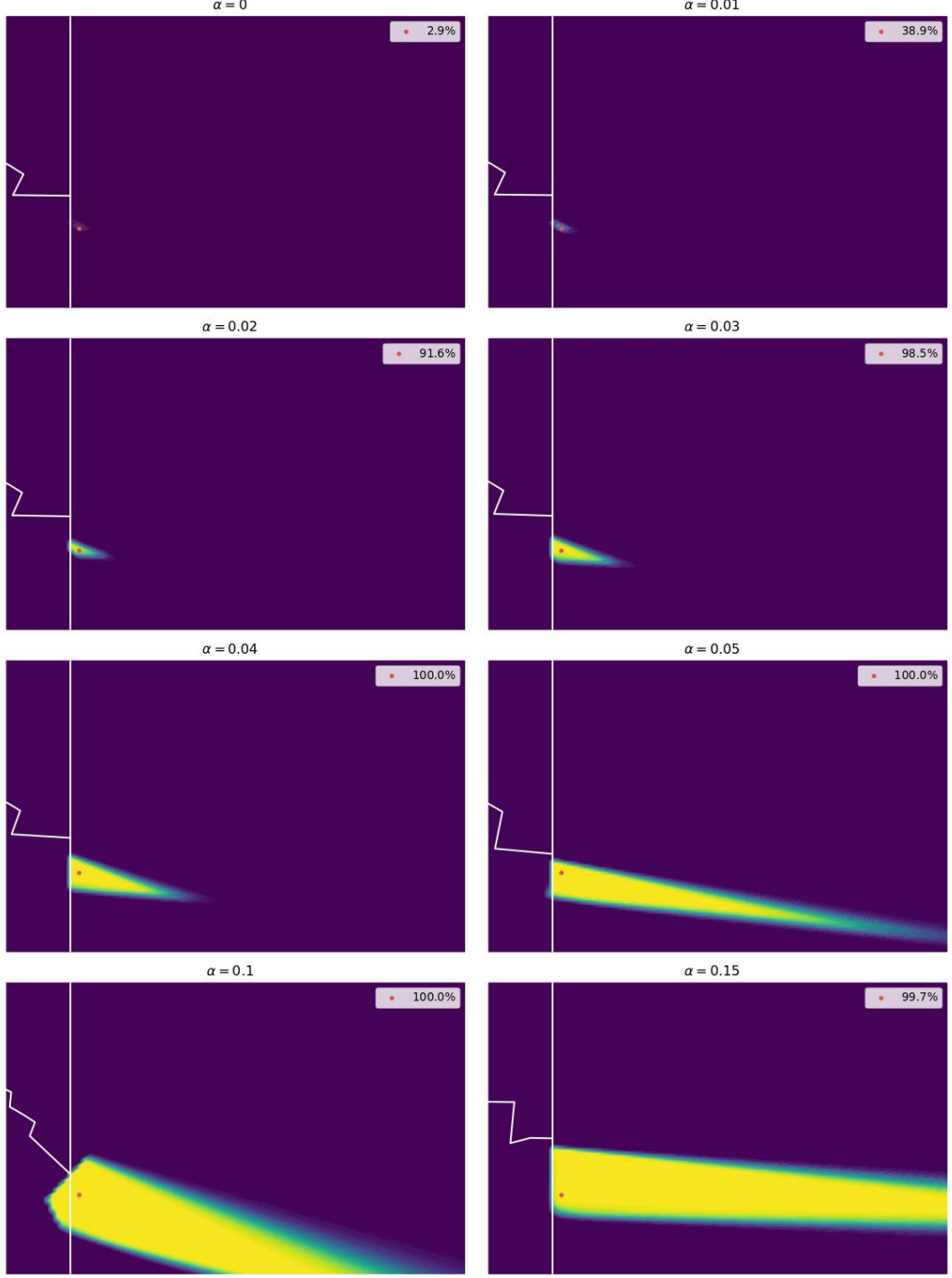
Robustness to imperfection

$$\boldsymbol{p}_i^{\epsilon} \sim \boldsymbol{p}_i + \mathcal{N}(0, \epsilon^2)$$

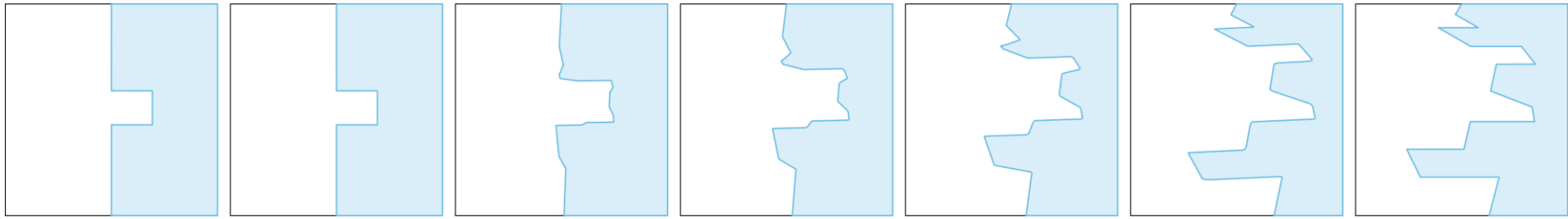


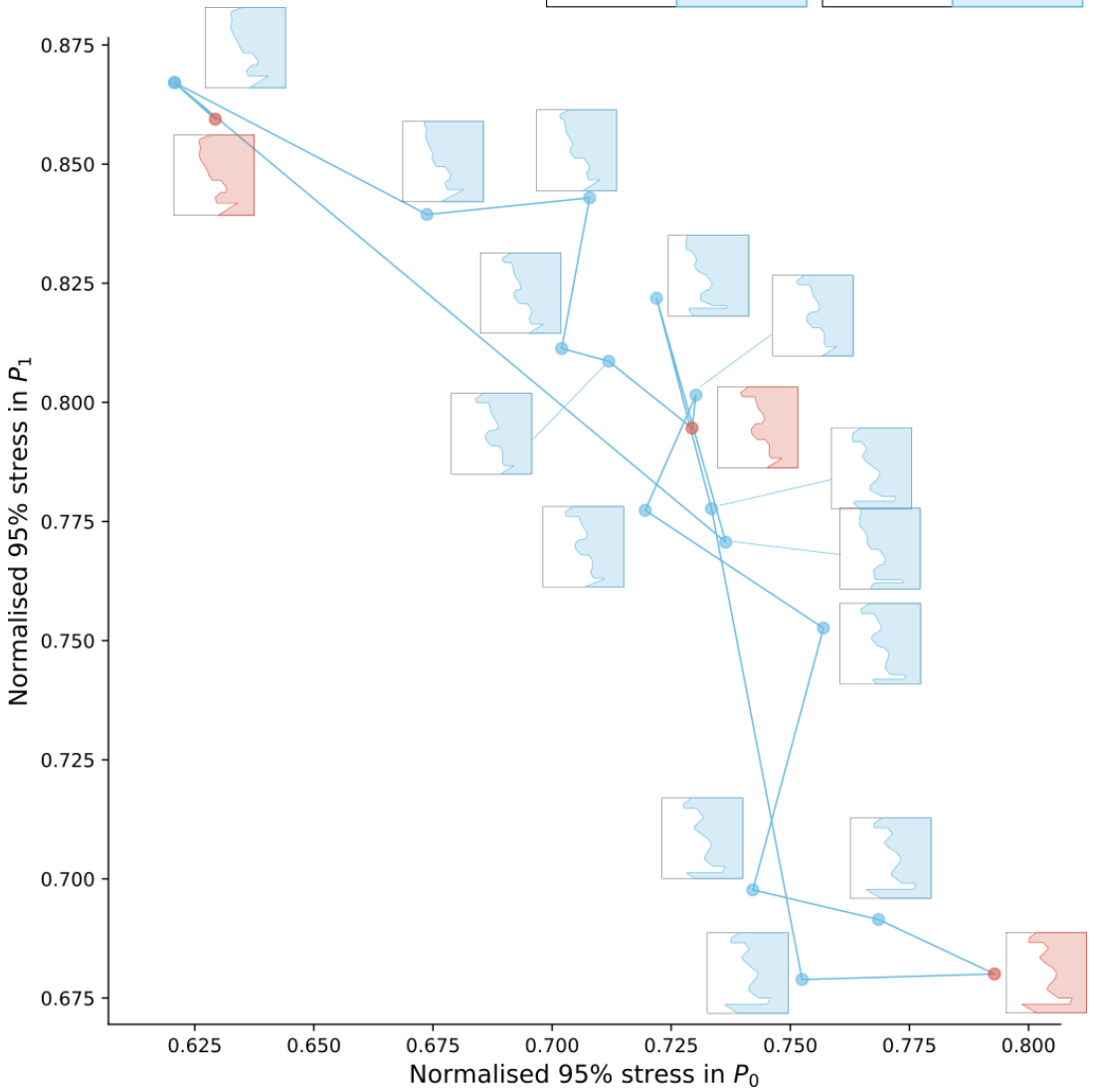





$\alpha = 0$
2.9%
$\alpha = 0.01$
38.9%
$\alpha = 0.02$
91.6%
$\alpha = 0.03$
98.5%
$\alpha = 0.04$
100.0%
$\alpha = 0.05$
100.0%
$\alpha = 0.1$
100.0%
$\alpha = 0.15$
99.7%

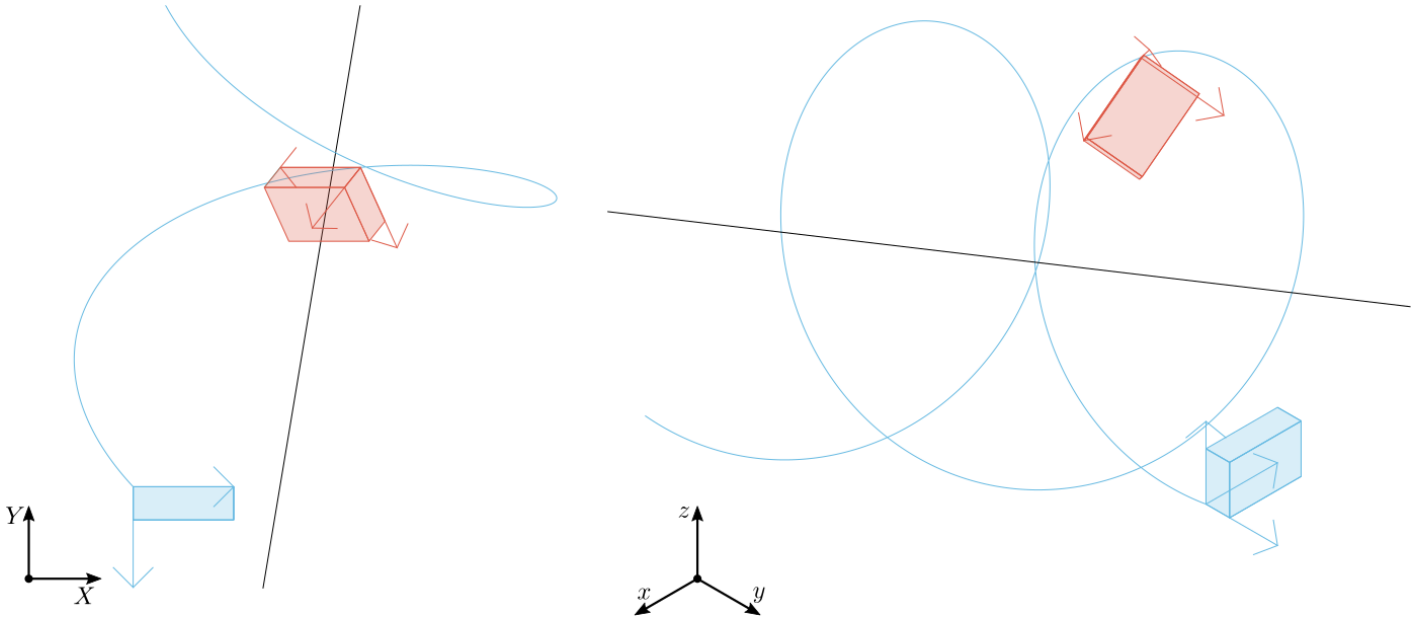
Interpolation

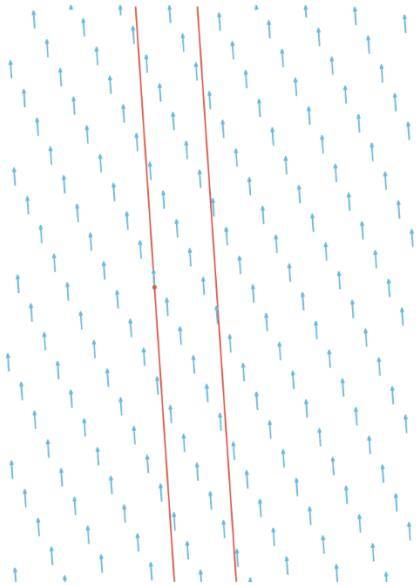


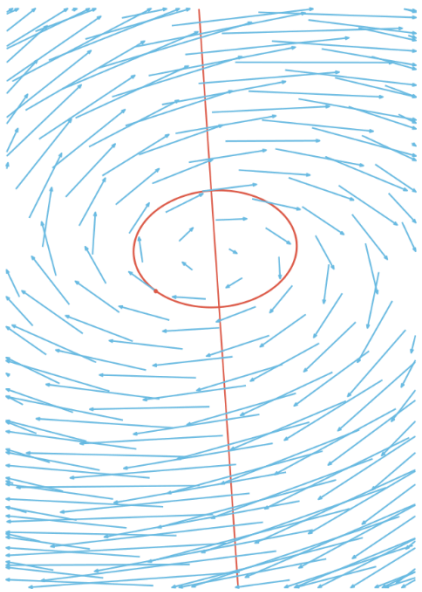


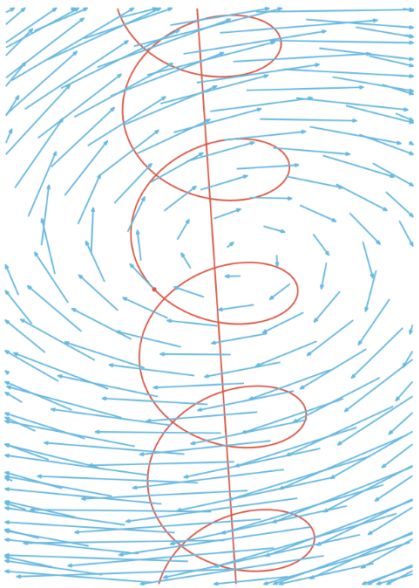
3D - Theory

Unit Dual Quaternions







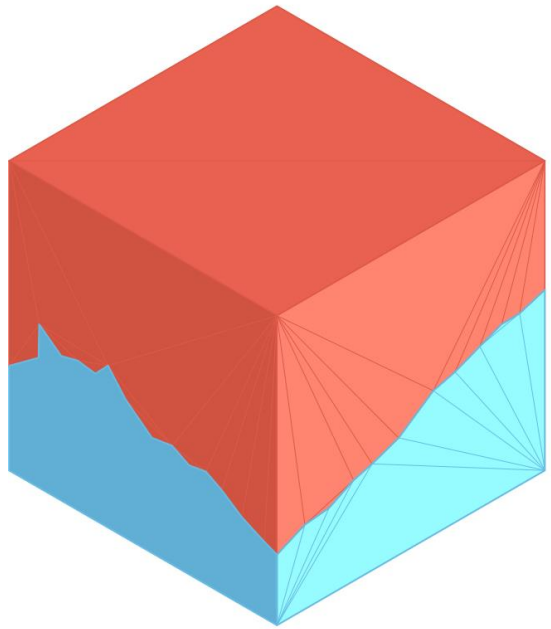


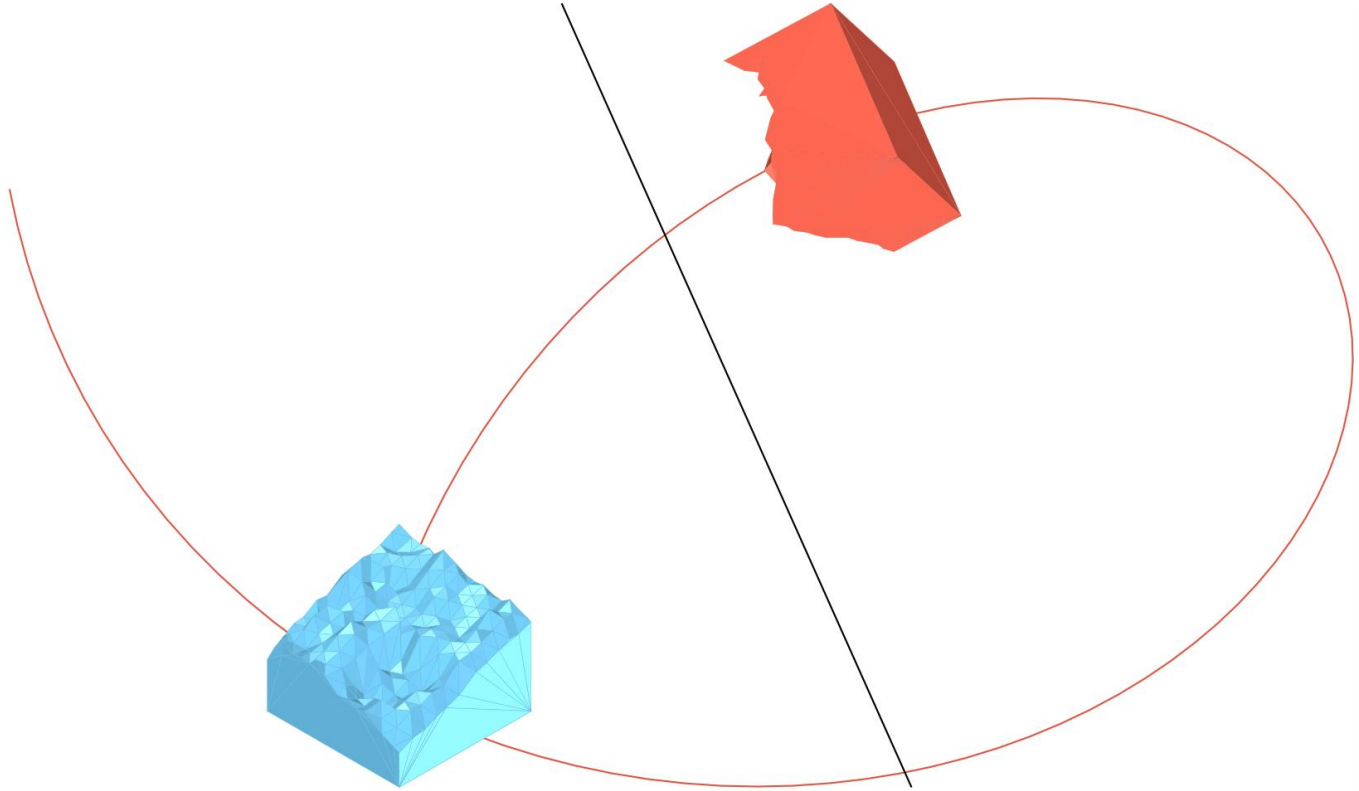
$$\hat{q} = \left[\cos\frac{\hat{\theta}}{2}, \hat{\boldsymbol{u}}\sin\frac{\hat{\theta}}{2}\right]$$

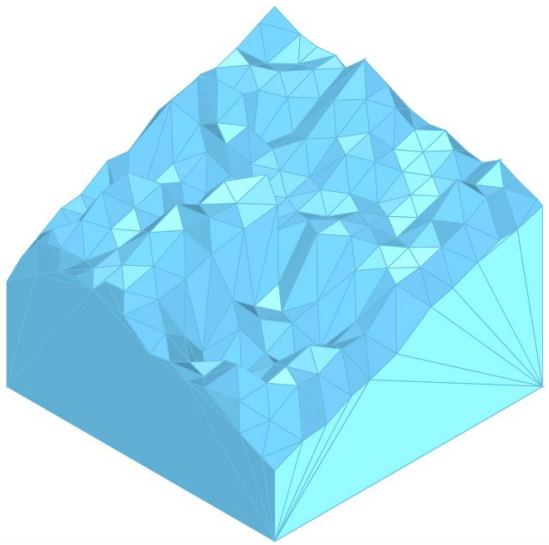
$$\hat{\theta} = \theta_0 + \epsilon\theta_\epsilon$$

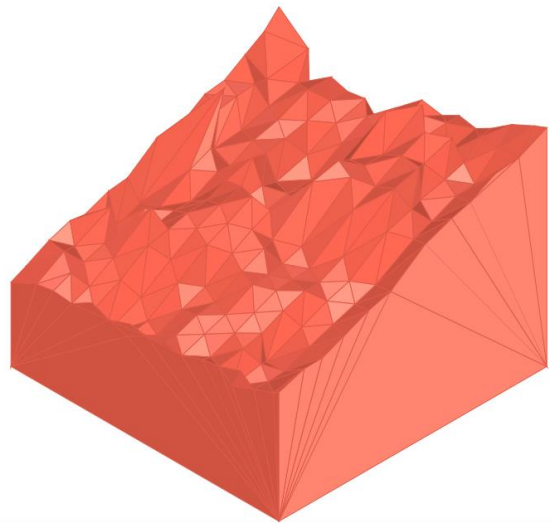
$$\hat{\boldsymbol{u}} = \boldsymbol{u}_0 + \epsilon\boldsymbol{u}_\epsilon$$

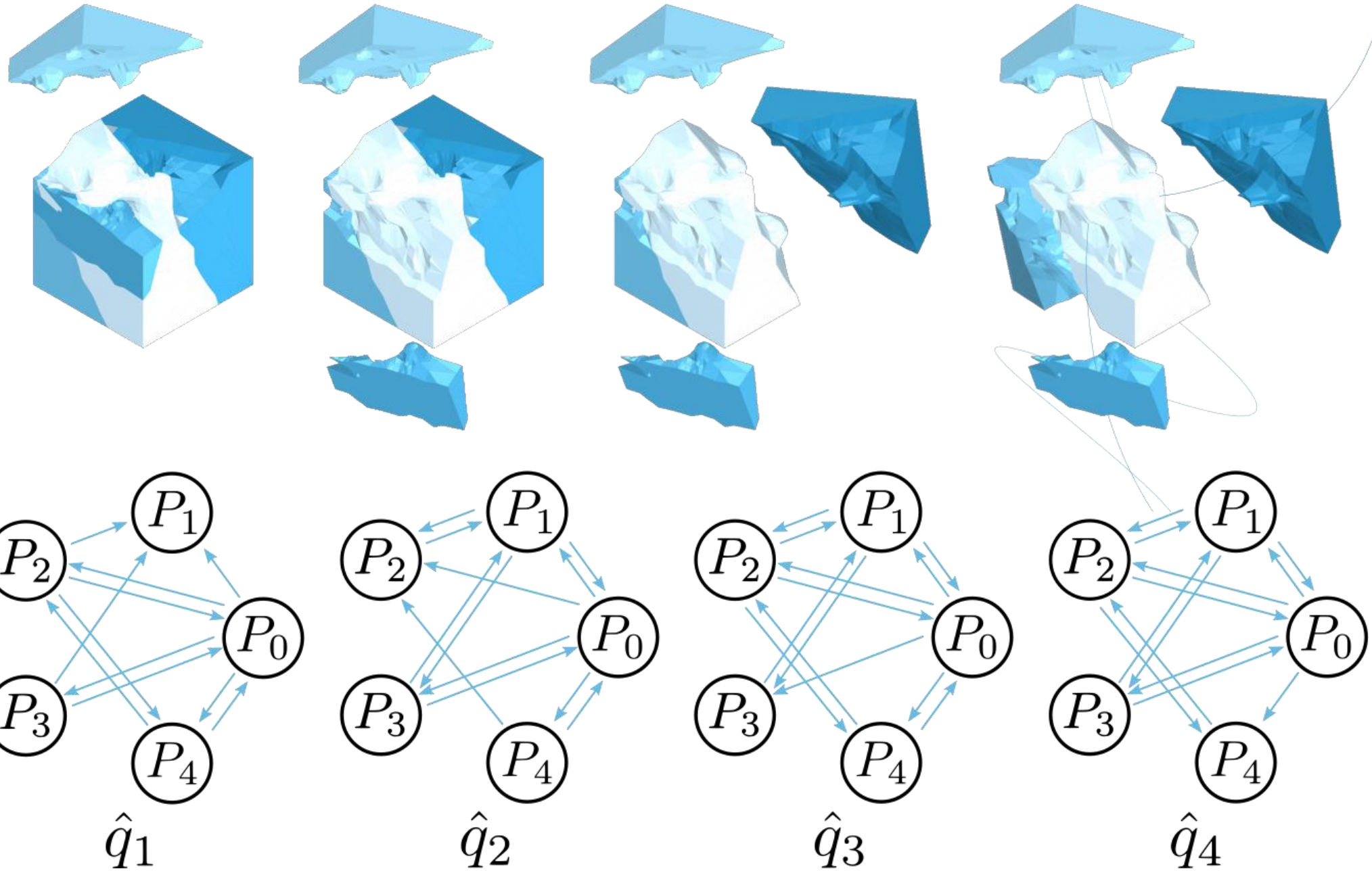
$$\epsilon^2 = 0$$



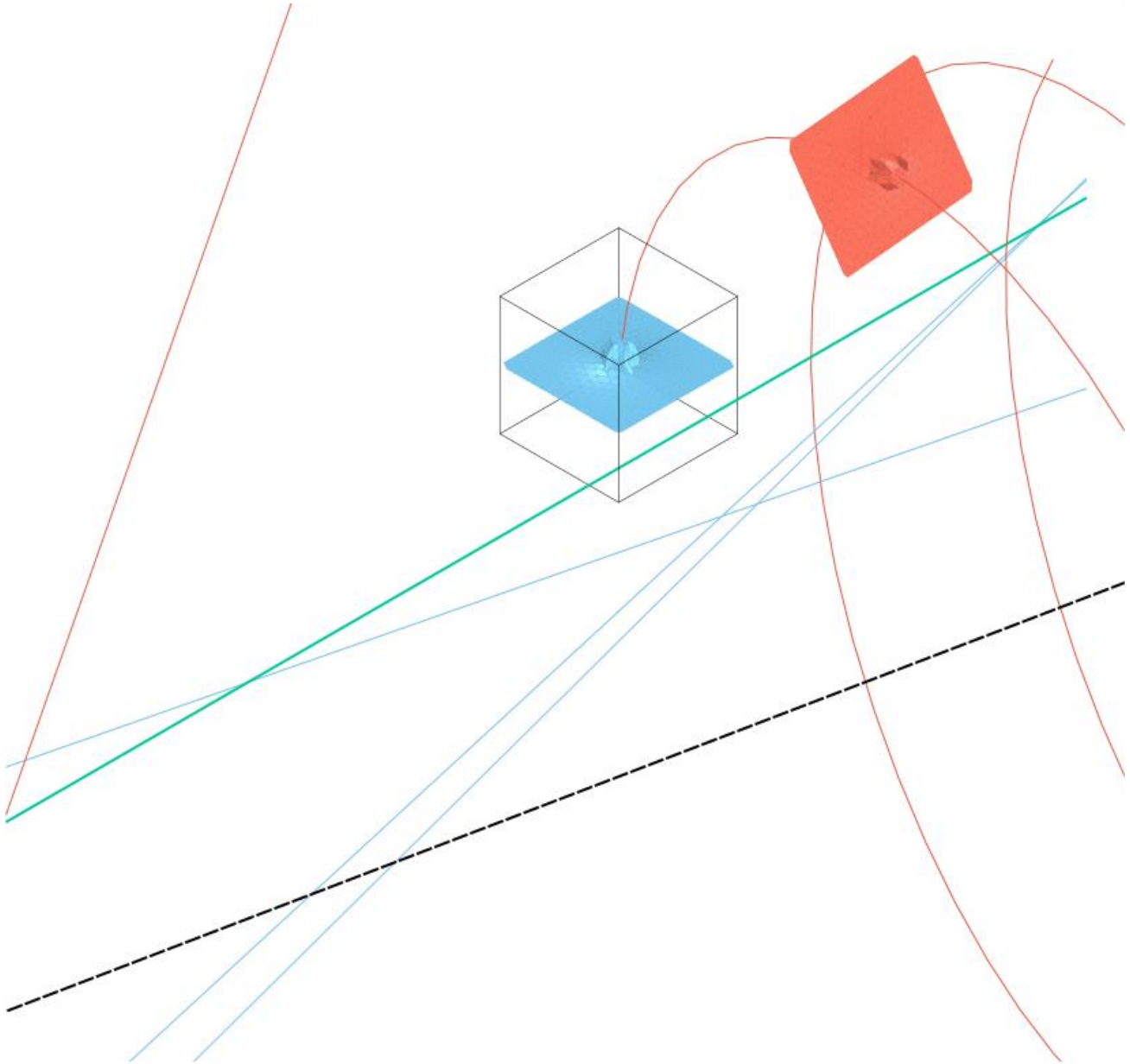







P_1
P_2
P_0
P_3
P_4
$\hat{q}_1$
$\hat{q}_2$
$\hat{q}_3$
$\hat{q}_4$

2D - Results

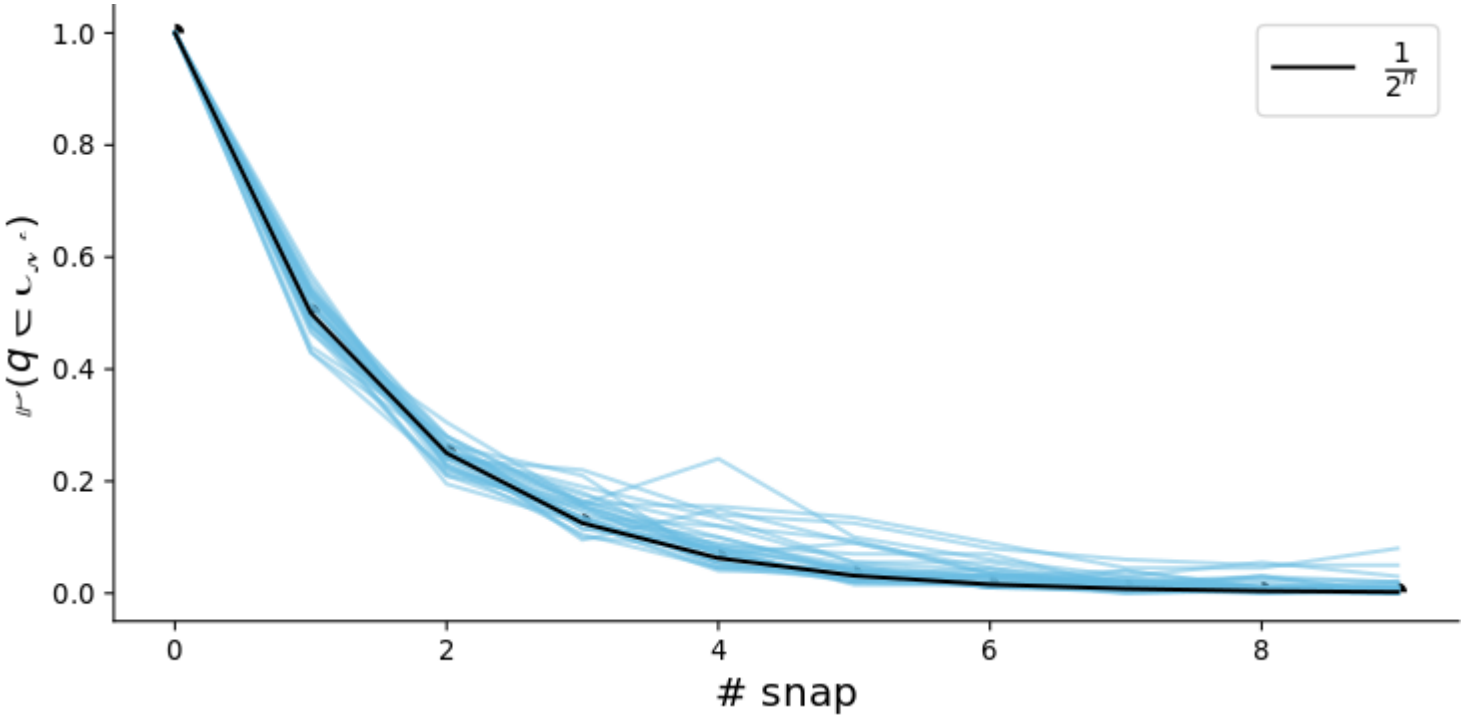


$$\begin{pmatrix} A_{\mathcal{M}} \\ -\tilde{\boldsymbol{x}}^T \end{pmatrix} \boldsymbol{x} \geq \begin{pmatrix} 0 \\ -||\tilde{\boldsymbol{x}}||^2 \end{pmatrix}$$

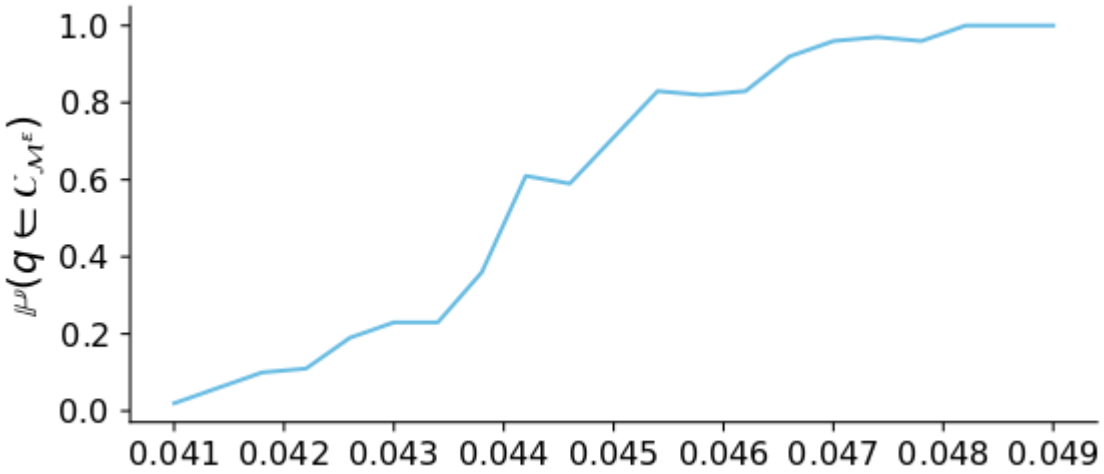
Cone of freedom

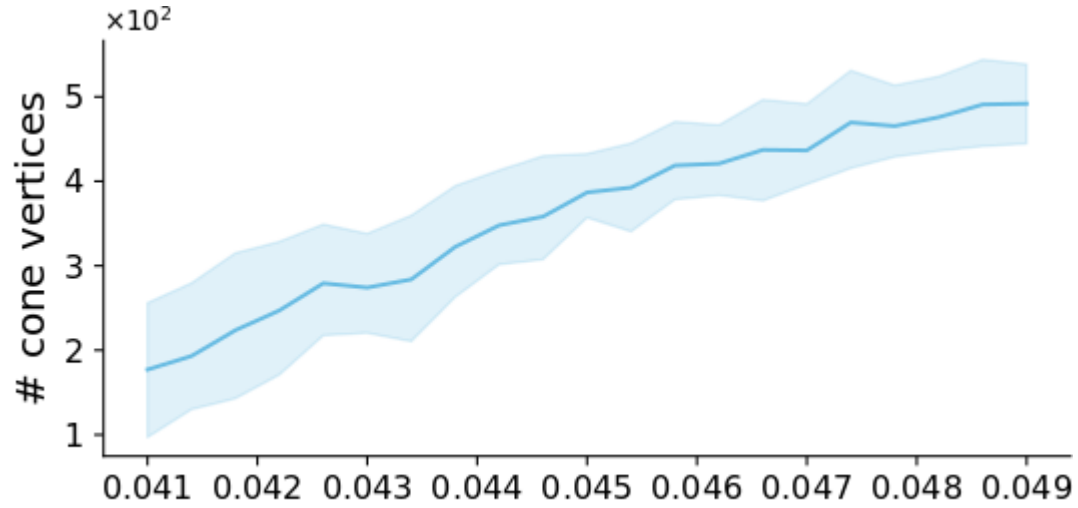
Robustness to imperfection

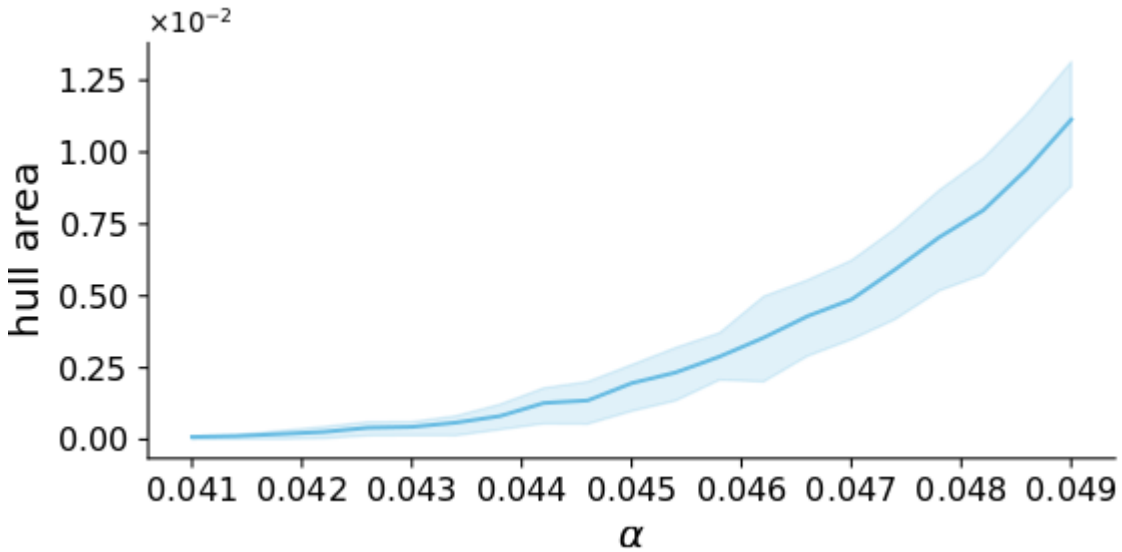
$$\boldsymbol{p}_i^{\epsilon} \sim \boldsymbol{p}_i + \mathcal{N}(0, \epsilon^2)$$

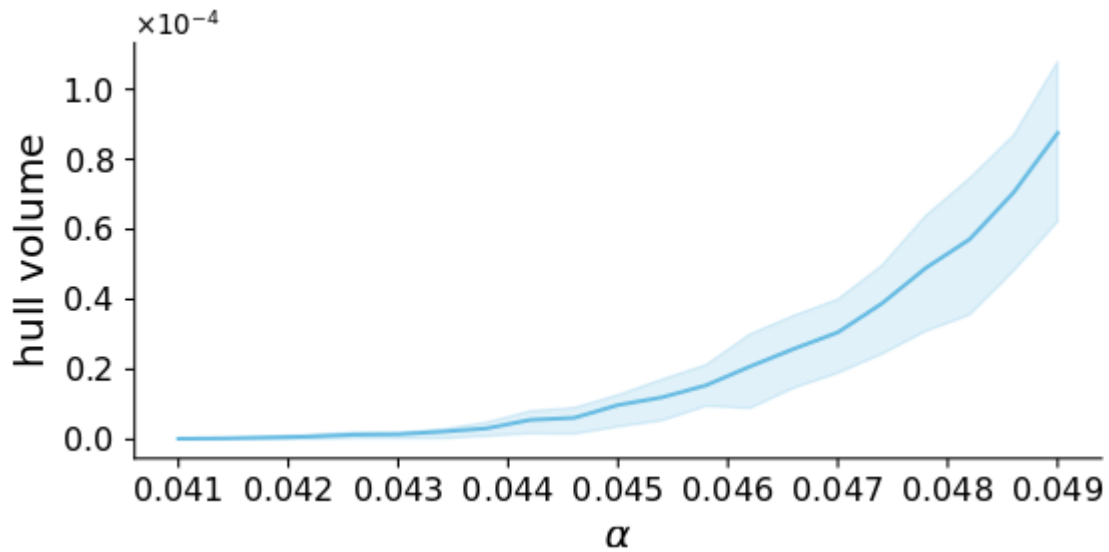


$$\forall f \in F \; \forall \boldsymbol{v} \in V(f) \; \boldsymbol{m}(\boldsymbol{v}, \hat{q}) \cdot \boldsymbol{n}_f \geq \alpha$$









Thank you